DOCUMENT RESUME

ED 223 672 TM 820 797

AUTHOR Leino, Jarkko
TITLE Psychometric Test Theory and Cognitive Processes: A Theoretical Scrutiny and Empirical Research. Research Bulletin No. 57.
INSTITUTION Helsinki Univ. (Finland). Inst. of Education.
REPORT NO ISBN-951-45-2450-0; ISSN-0073-179X
PUB DATE Oct 81
NOTE 84p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS Aptitude; Cognitive Ability; *Cognitive Processes; *Cognitive Style; *Individual Differences; *Mathematics Achievement; Performance Factors; Psychometrics; Secondary Education; *Task Analysis; *Test Theory

ABSTRACT

This report is the third in a series of research projects concerning abilities and performance processes, particularly in school mathematics. A theoretical scrutiny of traditional psychometric testing, cognitive processes, their interrelationships, and an empirical application of the theoretical considerations on the level of junior secondary school comprised the current study. Psychometric testing has been used to reveal mental processes demanded in task performance. It has been noted that most tasks can be executed by the examinees in several manners. Therefore, factorizing test scores may not reveal basic dimensions of performing processes. Cognitive theory focuses on very basic processes underlying test performance. Thus far, the research results of cognitive theory have been primitive when compared with the conceptual level of school instruction. Cognitive processes were described in terms of relevant level of school instruction in the empirical section of the study. Mathematical thinking and performing processes were well described by the system developed for the study. Results indicated that the processes used in the study explain most of the variances of "primary" abilities and achievements in mathematics. These processes also reveal valuable information about individual differences of the students. (Author/DWH)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED223672

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

J. Leino

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

RESEARCH BULLETIN

Institute of Education

University of Helsinki

SF-00100 Helsinki 10

Finland

No. 57

October 1981

Jarkko Leino

PSYCHOMETRIC TEST THEORY AND COGNITIVE PROCESSES: A THEORETICAL SCRUTINY AND EMPIRICAL RESEARCH

TM 820 797

Leino, Jarkko: Psychometric Test Theory and Cognitive Processes: A Theoretical Scrutiny and Empirical Research. Research Bulletin No. 57. Institute of Education, University of Helsinki. 1981. 74 pp.

The present study consists of a theoretical scrutiny of traditional psychometric testing, cognitive processes, and their interrelationships, and an empirical application of the theoretical considerations on the level of junior secondary school. The context of the empirical part was offered by school mathematics.

Psychometric testing has been used to reveal mental processes demanded in task performing. It has been noticed that most of the tasks can be carried out by the examinees in several ways. Hence, factorizing test scores does not necessarily reveal basic dimensions of performing processes.

Cognitive theory focuses on the very basic processes underlying test performances. So far the research results of cognitive theory have been rather primitive as compared with the conceptual level of school instruction.

In the empirical part of the study an attempt is made at describing cognitive processes in terms of relevant level of school instruction. The system developed describes well mathematical thinking and performing processes. It was found out that the processes used in the study explain most of the variances of "primary" abilities and achievements in mathematics, and, in addition to that, brings valuable information about individual differences of the students.

RESEARCH BULLETIN

Institute of Education, University of Helsinki
Head: Erkki A. Niskanen, Professor of Education
Fabianinkatu 28, SF-00100 Helsinki 10, Finland

Jarkko Leino

PSYCHOMETRIC TEST THEORY AND COGNITIVE PROCESSES: A THEORETICAL SCRUTINY AND EMPIRICAL RESEARCH

Helsinki 1981

Preface

I wish to express my deepest gratitude to the United States Educational Foundation in Finland for the grant I received to do research at Stanford University and the University of Oregon. Of my American colleagues I owe a special debt of gratitude to Professors Alan Hoffer and Oscar Schaaf. The inspiring discussions and seminars we had were very valuable for my work.

I thank Associate Professor Anna-Liisa Leino for the invaluable help with linguistic problems.

I am grateful for the inclusion of the study in the Research Bulletin series of the Institute of Education.

July 1981
Jarkko Leino

Table on contents

1. The purpose of the research 1
2. The development of psychometric testing 2
2.1 Early decades 2
2.2 Developmental directions 5
2.3 Approaching processes 7
3. Cognitive processes 11
3.1 A new approach 11
3.2 Memory structure and functioning 12
4. Tentative combining of processes and aptitudes 16
4.1 Cognitive components 18
4.2 Sternberg´s theory 25
4.3 Carroll´s approach 27
5. Cognitive styles 28
5.1 Some dimensions of cognitive styles 30
5.2 Cognitive styles in mental structure 35
6. Empirical approach 37
6.1 Framework of the empirical study 39
6.2 Subjects and procedure 40
6.3 Tests and scoring 41
7. Results 44
7.1 "Primary" abilities and attitudes 44
7.2 Information-processing abilities 46
7.3 "Primary" abilities and information-processing abilities 48
7.4 Good, average, and poor achievers 52
8. Discussion 55
References 59
Appendices 68

Motto:"You have your crusaders, you have religious enthusiasts, you have your desires for liberty, you have all the other patterns, but you've got to go further back still. Further back to a hinterland. Behind materialistic results there are ideas. Visions, dreams ... Visions can open new worlds to you - and visions can also destroy the worlds that already exist." (Agatha Christie)

1. The Purposes of the Research

This report is a continuation of a previous research by the present author concerning abilities and performance processes especially in school mathematics. The first report of the research project consisted of a review of related studies and an empirical recapitulation of mathematical abilities and attitudes. The abilities were measured by means of traditional psychometric tests and the attitudes by means of an attitude inventory (Leino 1977). The second report was concerned with mathematical thinking and performing processes of eight grade students.

The main purpose of the second research was to develop a description system for mathematical processes and evaluate the quality of the system (Leino 1978). Neither a detailed analysis of the relationships between abilities and processes nor a comparison of typical profiles of students were carried out.

This third report consists of a theoretical scrutiny concerning the development of the research of human abilities and cognitions and an empirical application in the context of school mathematics. In order to keep empirical research rather close to school education the process variables have been formed on the relevant level. In the technique used, structural groups of mathematical problems and situations were presented to the subjects. They were asked to perform all their thinking processes aloud, which were tape recorded. They had also paper to draw pictures on, to help and to write necessary calculations. The primary data is the same that gathered in the previous stages of the project but further analysed here.

A purpose of the study is to find out what kind of information thinking and performing processes give, in addition to that given by psychometric tests, i.e. the purpose is to find the relations between these two sources of information about abilities or capabilities and their functions. It is

obvious that psychometric tests measure abilities but, whether or not they measure mental processes whose final products test performances are, is not so obvious. For instance, the planning of a solution of a problem is likely to be excluded when only tests are used and, especially in the problem-solving process, the initial planning is often in key position. However, abilities are in close connection with the actual processes, particularly when test items are small tasks that cover the main phases of performing typical mathematical tasks.

Another purpose is to discover the characteristics of the profiles of the different subjects in terms of abilities and cognitions. Especially interesting is the comparison of the profiles between successful and unsuccessful students because that indicates the kind of necessary corrective activities that would have to be offered and is highly relevant from the view-point of instructional objectives.

The third purpose of this report is to lay a theoretical basis in terms of modern cognitive theory for the future research parts of the profect in which the problem-solving strategies and teaching them will be studied, in particular.

2. The Development of Psychometric Testing

2.1 Early Decades

It is suprising how short a time mental testing has been used in psychology and education. Late in the 19th century when general measurement psychology (e.g. Ebbinghaus and Weber) and differential psychology (e.g. Galton and Binet) had started to develop and the idea of compulsory school attendance had become general, Alfred Binet was commissioned by the French government to study children's ability to profit from education (Charlesworth 1976). The purpose of Binet was to identify different levels of mental ability in children - to identify inferior states of intelligence, in particular, - in order to facilitate optimal development in each child.

Before Binet, the concept of intelligence was not clearly differentiated from other concepts, such as soul, mind, and consciousness. The exact formula of correlation coefficient was not proposed by Pearson until 1896, even though Galton had used correlation in terms of regression ten years earlier and among many mathematicians (e.g. Laplace and Gauss) the formulation of correlation was only a matter of time. During the following decade, after Pearson's publication, practically all basic correlation methodologies (e.g. standard deviations, multiple correlation, regression theory, and error estimates) were developed by Pearson and his associates in England.

The idea of factor analysis which is mainly used for identifying the basic dimensions of mental abilities was presented by Spearman in 1904. He also introduced the correlation coefficient as an index of the reliability of measurements (but did not use the term reliability at that time). He used the correlation for attenuation, i.e. for estimating the amount of correlation that could be freed of error. The reliability theory was further developed by Spearman six years later and, independently, at the same time by Brown (hence Spearman-Brown formula) concerning the effect of the test length on the reliability of measurements. This initiated mental test theory, which new field had prominent developers, Thorndike in particular.

Ever since the creation of the concepts of human ability or intelligence there were many discussions concerning the possibility of intelligence being a hereditary quality. The early representatives of genetics could not accept that, because intelligence was multifactorial, not a single trait, such as eye color, for example. By experimenting with rats, it could be proved that at least part of animal intelligence was herited. Discussions of human intelligence have been going on ever since.

The other controversy concerning human ability (abilities) originated in the early work of Spearman (1904). "All branches of intellectual activity have in common one fundamental function, whereas the remaining or specific elements of the activity seem in every case to be wholly different from that in all others" (Spearman 1904, 284). This bifactor theory of general ability (g) and specific ones (s's) was largely accepted for a long time. Spearman himself identified two specific abilities, viz.verbal ability and fluency.

It was Thustone who questioned the concept of general g. He studied intensively the mathematics of factor-analysis which had long been well-known to mathematicians but only to very few psychologists and he could generalize the factor model of Spearman. Some smaller researches of his were published earlier in the 1930´s but in 1936 he, together with his wife and their students, applied this new model to a major data of human ability. The results, published in 1938 (Thurstone 1938) showed no evidence of a general factor but, instead, several group factors were identified. These "primary mental abilities" consisted of perceptual speed (P), verbal ability (V), word fluency (W), memory (M), numerical ability (N), general reasoning (R), inductive reasoning (I), deductive reasoning (D) and spatial-visualizing ability (S).

The group factor was rapidly accepted by psychometricians in the late 1930´s. Many Technical ideas of Thurstone´s factor-analysis were far-reaching: common factor variance, communality, rotation to a simple structure and oblique rotation. Thurstone used the technique of oblique rotation while identifying his "primary" abilities. It is this technique that obliterates the general factor and gives a great emphasis on tasks that correlate with each other, even though the correlation coefficients may be low. If the orthogonal technique is used on Thurstone´s data, it generates a general factor with 31 % explanation of the variance and some eight specific factors with less than 7 % explanation of each (Eysenck 1939). The "primary" abilities of Thurstone correlated with each other and the second order factors show the common parts of the group factors. In any case, the influence of Thurstone´s researches was powerful not only because of the interesting results but also on account of the great skill showed in test construction.

The validity of the mental tests was computed by correlating the scores of the tests with other measures presumably reflecting the same abilities. Correlations reported were usually rather high. The split-half reliabilities were often very high, which can partly be explained by the speed element of the tests (Carroll 1978).

The early ability tests were suggested to have a whole variety of educational use: homogenous grouping of students, aid for teachers (e.g. a student with a high test score and low school performance was considered

to need extra help), prediction of future school success, career selection, admission to higher institute or vocational job.

The general notion that the ability test would measure a kind of inborn intelligence independent of training and schooling was criticized very early, for example, by Brigham in his article in 1930 (cited in Carroll 1978, 43) as follows: "The test scores very definitely are a composite including schooling, family background, familiarity with English, and everything else, relevant and irrelevant." A serious disadvantage of the tests of "primary" abilities was also the fact that the actual significance of different abilities in certain school subjects as well as school learning in general was very difficult to identify or to prove.

2.2 Developmental Directions

Though "the primary abilities" were accepted as the main stream in the 1930´s and 1940´s the theory of general ability and specific abilities had also supporters in different flexible forms (e.g. Thomson, Burt, Vernon, and Cattell). This flexibility is to be seen in the fact that many researchers included the group abilities as an intermediate stage between general factor or factors and specific abilities (Burt 1949; Vernon 1950; Cattell 1963). For instance Vernon describes a hierarchical model of abilities, consisting of general g at the top, two major group factors (verbal-educational v:ed, and spatial/kinesthetic-mechanical k:m) and many special abilities, such as reading, spelling, creative, psychomotor, and mathematical abilities. Cattell distinguishes two general abilities namely fluid and crystallized intelligence (g_f and g_c). The factor g_f expresses individual perception ability of complex relationships and g_c the judgements of individual thought. The former concerns perception of, and logic in spatial, visualizational, and inductive relations measured by culture-free tests. The latter tends to be close to traditional tests of verbal, general reasoning and numerical abilities. Cattell emphasizes that the inherited part of intelligence will rather be seen in g_f than in g_c while the reverse is true of the acquired part of intelligence. The most important feature in Cattell´s approach, from the view-point of modern ideas, was the attempt to see performances,

in the sense of processes, as fuctions of abilities.

Another direction of test analysis in which reasons for individual differences were in focus was created by Thorndike (Thorndike et al. 1926). He tried to sophisticate the concept of mental performance by distinguishing three different aspects: 1) altitude which refers to the degree of difficulty of the tasks that an individual can do successfully, 2) width which means the number of tasks of a given difficulty that an individual can do, and 3) speed which is the required time to accomplish a task correctly. In "classical" test theory the task difficulty is measured by the percentage of correct answers. The total score of a test is simply the sum of correct answers in spite of the fact that different tasks (items) are different as regards their difficulty. This method usually produces an index of item difficulty which is highly biased by ability in the sample. The better models where task difficulty is not biased by the ability distribution in the sample, such as latent trait model (Lord 1952; Rasch 1961; Lord and Novick 1968; Lord 1970), have been adopted very slowly. These models give a possibility to fit the test according to some individual property such as previous success (tailored testing) and get equal test alternates. These ideas have lead to the development of the so-called item banks.

The question of group abilities has stimulated the development of the techniques to construct consistent tests. One measure of internal consistency of the tasks of a test was formulated by Kuder and Richardson (1937) (Kuder-Richardson K-20 test) and a generalization of it was suggested by Cronbach (1951) (α- coefficient). Though these coefficients of consistency reliability give one basis for test homogeneity, the whole question has proved to be more complicated than expected. For instance, any traditional item analysis or the high value of the coefficient alpha does not guarantee that the test measures only a single ability.

The most usual technique to suggest and determine that a test has several dimensions is factor-analysis. Unfortunately, this technique has been and still is susceptible to subjective interpretations. The main problems are: how many factors should be accepted in the common-factor space, how to estimate the communalities, which one is the most adequate the orthogonal or oblique rotation, and what name should be given to a certain factor.

Many of these problems can be solved on the basis of some objective criterium but there are a number of objective techniques to select from. Fortunately, practically all objective techniques lead to rather similar results.

The problem of the number of "primary mental abilities" cannot be soved objectively. Even Thurstone (1938) realized that it is possible to get an indefinite number of group factors by including a sufficent number of similar types of tasks in the test. The problem is, how many of these factors are of substantial importance for education, life or other respectable purposes. For instance in school mathematics some 6-7 factors explain 70-80 % of the variance of school achievements. There are certainly samples (school classes etc.) where the explanation is less dependent on sample selection, achievement criteria and the like.

2.3 Approaching Processes

The significance of intelligence has never been great for education. In terms of "primary" mental abilities intelligence might have greater usefulness if the student's weak abilities could be easily identified and improved by special programs. Unfortunately, very few programs of this kind have been developed and fewer still researched. One cause of this unfortunate state of affairs has been the general preconception of the immutability and inherited nature of abilities. Another cause is the vagueness of the concept of intelligence and the lack of knowledge of the underlying processes.

One of the researchers of mental abilities emphasizing the underlying processes was Thomson in the early 1920's. He proposed that a mental task should be considered as a sample of mental operations. The correlation between two tests is a consequence of the overlapping sets of operations (Brown and Thomson 1921).

It was Guilford who, following the ideas of Thurstone, presented the first clearly process-oriented model of intelligence (1967). He had noticed that the oblique rotation used by Thurstone gives undue importance to tests

which actually correlate rather little with each other. He decided to use orthogonal rotation and try to get a simple structure. While carrying out the orthogonal rotation of"primary" mental abilities he found that Thurstone´s 7 or 13 abilities were not an exhaustive system. Thus Guilford proposed a comprehensive theoretical model for a structure of intellect (SI model) with three principal dimensions: 1) mental operations with the subcategories of cognition, memory, divergent production/thinking, convergent production/ thinking,and evaluation; 2) content (subject material) with the subcategories of figural, symbolic, semantic,and behavioral ones; and 3) products (forms of information) with the subcategories of units, classes, relations, systems, transformations,and implications. Because of independence, the dimensions are orthogonal and the total number of possible abilities is thus 5x4x6 = 120. Then Guilford started to fill the cells of SI model with large test batteries and with the aid of factor analysis.

The fundamental weakness of Guilford´s model was the validity: fitting data to hypothesis will not yield information on the validity of the model itself. It is very easy to see that, e.g. the categories of product and operation dimensions are at least partly hierarchically or otherwise logically ordered and thus the demand of uncorrelated factors is impossible. There are also other smaller weaknesses such as the narrowness of the concept of cognition, the lack of numerical ability etc. (Carroll 1968). However, Guilford has clearly proposed a new idea to the theory of human intelligence while adapting perception and memory processes as parts of intelligence and attempting to combine many well-known results from both psychometrics and experimental psychology.

Very soon Guilford´s ideas attracted the attention of other researchers. For instance, Cattell (1971) presented his model organized on the basis of three dimensions, too: 1) action phases (involvements of input, internal processing and storage,and output), 2) content (experiental-cultural and neural-organizational), and 3) process parameters (complexity of relations to be educed, memory storage, retentivity, retrieval, and speed). It can be seen that the dimension of content consists of two general abilities g_c and g_f previously dealt with.

An important addition to earlier models was suggested by Jensen (1970) who

distinguished two levels of abilities: I simple associative learning ability and memory where the information is neither transformed nor mentally manipulated, and II higher order conceptual learning and problem solving where the input information is mentally manipulated. Jensen regarded intelligence as a synonym of learning ability which was not very successful because learning is only one face of the concept. In identifying the first level ability he used tasks of serial learning and memory for paired associations and the second level ability tasks of Raven's coloured progressive matrices and figure copying, thus clearly process-oriented tasks.

All process approaches presented above were good attempts to consider abilities as processes or competencies or at least to interpret psychometric test results in terms of functions of abilities. No actual quantitative index of processing such as learning, concept formation and problem solving were presented. Still, there was a need of a methodology and theory for interpreting psychometric tests as cognitive tasks and characterizing factor-analytic abilities accoring to a model of cognitive processes (Carroll 1976).

The abilities are usually identified in a representative collection of tasks by factor analysis. The performances have to be considered as end products of and thus without clear connections with corresponding mental processes. The data used consists of groups of items each of which is only believed to measure a single ability in the traditional sense (consistency). From a practical point-of-view a test has been composed of numerous items, each of which can be completed by at least some examinees in a rather short time. This characteristic makes the test somewhat artificial; neither in school nor real life are the problems confronted so small, uncomplicated, and reflecting only a certain competence or competences. In traditional test theory each item should give a reliable measure of some typical performance, a group of items a discriminating set of reliable measures, and the groups together a representative coverage of competences. Because the ability to solve complex problems often implicates more than the collection of abilities each implicated in separated parts of the same problems the validity of psychometric tests is not complete.

The aspects mentioned above are particularly significant from the viewpoint of successful teaching. The teacher has to develop correct thinking and performing processes in the students. That is why the abilities must be

regarded as process-oriented and in close connection with processes. In this light the intelligence composed of different abilities,such as verbal ability in the sense of language competence, general reasoning in the sense of ability to manipulate concepts and relations, inductive reasoning in the sense of ability to find out a rule governing individual cases and to use the rule, and spatial ability in the sense of ability to perceive and compare spatial figures or other visual stimuli, has great importance for school instruction and the demands of different fields of life. Naturally the abilities are not to be seen as fixed or inherited but as learnable and developing through different experiences, most systematically in school.

Before the introduction of cognitive theory Piaget has to be mentioned. His theoretical formulations are based on semi-experimental researches without complete control or standardization. He was neither a traditional psychometrician nor an experimental psychologist. He emphasized the contribution of genetics to cognitive ability but on the basis of biological organization throughout one's life time. The emerging cognitive structures are increasingly complex and qualitatively different in the consequent stages of development. While psychometricians attempted to quantify the amount of genetic factors in explaining the individual differences, Piaget considered the emergence of qualitative cognitive functions in the course of the individual's development. According to Piaget cognitive growth is accounted for by such basic processes as assimilation and accommodation. These processes are dynamic, changing and well-balanced. Organism is continuingly interacting with new demands and situations, assimilating new information and accommodating itself accordingly. Piaget was never interested in individual differences or the possibilities to facilitate the growth of the individual by means of education. Recently some efforts have been made to produce testing instruments, first, to identify Piagetian developmental stages and then to analyse the individual abilities (in terms of variance) on each stage (see Tyler 1976). Piaget attached great importance to structure, the main characterictics of which are as follows: a) wholeness (totality of the interdependency of elements), b) transformation (constructive aspects of the structure), and c) self-regulation (compensatory activity of organism to outside influences) (Piaget 1970, 5). We can see that Piagetian structure is a dynamic concept.

3. Cognitive Processes

3.1 A New Approach

While psychologists have always had to tie psychological concepts firmly to behavior it was an argument of cognitive psychology that behavior is merely a manifestation or result of underlying thinking and that the most important concern should be with these underlying processes. The development of information theory (Shannon and Weaver 1949) and artificial intelligence (see Simon 1979) since the early 1950's provided good impacts on describing and explaining thinking in detail, in terms of information processing. This new approach has started a Kuhnian (Kuhn 1970) revolution in cognitive psychology.

The human being is seen as constantly interacting with information about the environment - taking in, storing, manipulating, and using, in short, processing information. One basic assumption of this new approach is, that processing can be broken down into subprocesses. In terms of these subprocesses and corresponding memory structure, the complex intelligent behaviors, such as perceiving and recognizing stimuli, learning and remembering, and problem solving, could be described, understood, and predicted better than ever.

The new cognitive science has taken main ideas of former theories such as gestalt, association, meaningfulness and rule learning, given them a new frame of mechanism and interpreted former research results within this new information processing model. In fact, it is suprising to notice how many old researches of our century - in addition to new ones - have recently been repeated and reported as the first really scientific researches. However, information processing model gives a simple and clear explanation for many old ideas. Besides, some new results seem to give an essentially deeper level, than former theories, for understanding and explaining complex cognitive processes in particular. It seems that there are possibilities to make this new theory useful in instructional practice, too.

3.2 Memory Structure and Functioning

In terms of cognitive psychology the cognitive process includes a) sensation and perception where stimuli are recepted and recognized, b) learning where the input is encoded, c) memory where the input is retrieved, and d) thinking where perceived, learned and remembered information is manipulated in some purposeful manner, i.e. in order to give some manifest response. (All information does not go through the whole of this detailed system; information is often only perceived and temporarily encoded for a response). The associative theory emphasized the contingencies between stimuli and responses especially by conditioning. Skinner, for example, had developed an application of this for teaching purposes. Gestalt psychologists (e.g. Köhler 1940) tended to abandon associative theory as incomplete or irrelevant, emphasizing, instead, a perceptual analysis of situations or problems. Memory was regarded as a system which was governed by principles of organization and laws of perceptual grouping. It was Tulving (1962) who added a new dimension to the organizations of processes, namely that of functional utility. The rule learning of, e.g. Gagné (1962) and the meaningful learning of Ausubel (1963) are both theories of learning and thinking. Information-processing theory can be regarded as a generalization of the above mentioned theories. (For a more detailed comparison of these theories see e.g. Mayer 1977).

Memory system in information-processing theory has two feature categories: a) structural properties, and b) functional properties. The latter category includes, as an important part, the controlling processes. The model of memory system, first presented by Atkinson and Shiffrin (1968), consists essentially of three different stores of information corresponding to the processing stages: 1) sensory register where stimuli are first registered by senses (visual, auditory, haptic,...), 2) short-term memory (STM) into which the recognized part of registered information goes, and 3) long-term memory (LTM) which is the permanent store of information.

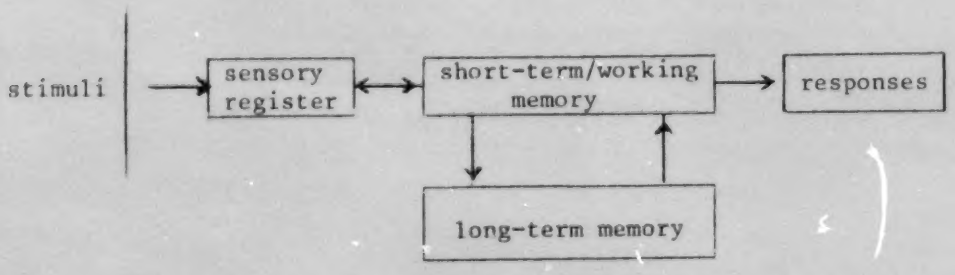


Figure 3.2.1 Schematic model of memory system

Since a large amount of information enters the sensory registers and only a limited amount of that can be prosessed at a time there is a need for a kind of control or limitation system. This concept is called attention.

Sensory register - or registers because each sense has its own register - serves the function of briefly holding information about stimuli. The information in register has much the same form as that which was initially presented. From sensory register the information gradually decays in a short period of time depending on the register. In iconic register the average duration after which the information can still be recognized is approximately 1/4 s (Haber and Standing 1970). There are circumstances where the iconic trace can stay longer, for several minutes (see Sakitt 1976). In echoic register the duration is more sensitive to experimental situation.

From sensory register the information transfers to STM through the complex process of pattern recognition. In this process the input trace in sensory register is extracted and compared to previously acquired knowledge of LTM and a pattern, e.g. a configuration of several elements that belong together, is said to be recognized when the trace or parts of that are equated with meaningful concepts in LTM. This process can also be partial. Only few recognized units can be distinguished in STM, usually 4-5 units but 7±2 at most (Miller 1956). These units are not necessarily in the form of sensory input. A unit can be a letter, number, familiar word or figure etc. The information in STM decays gradually in a period of say 15-30 s without rehearsal. Rehearsal is an active attention-demanding process where the units of STM are recycled over and over again through STM. Because it is STM where the recognized units are formed and manipulated (connecting them with relevant information from LTM), where rehearsal takes place, and where information can also be elaborated in other ways, STM is also called a working memory.

The LTM is essentially a permanent store for information. The information there is usually in an organized and meaningful form. The new information is encoded through STM into LTM as a part of already existing structure. The learning result may be a proper new piece in the semantic or visual structure or an episodic linkage about happenings. This learning process is not generally passive; one acts on information to elaborate it and relate it to previous knowledge and also the conditions of its occurrence. The retrieval is a similar,

active process, in which one tries to search previously encoded information that best matches the demands and purposes of the situation. Forgetting means the loss of ability to retrieve once-encoded information in LTM. In shorter periods of time forgetting can also refer to the loss of information in STM.

The model of two separate memory entities has been problematic. Not all researchers have accepted that (e.g. Wickelgren 1973). Nobody seriously believes that there exists two separate storages although brain studies have demonstrated that there is indeed a kind of partitioning of cortical function. Memory is conceived to be a large and permanent collection of neural components or nodes which become complexly and increasingly interassociated and interrelated through learning. Most of the nodes are normally passive and inactive and termed then LTM. The set of currently activated nodes is termed STM which is only a temporal state (Shiffrin and Schneider 1980, 24). When the activated nodes revert inactivated the information is said to be lost in STM. The activation of nodes can be caused by external or internal information input or by attention-demanding process. Some nodes are grouped so that the activation of one can activate also the others of the group.

The information in LTM is at least partly hierarchical. This means that the activation of certain nodes can activate others in one direction but not vice versa. An automatic process can be understood as a sequence of nodes that nearly always become active in response to a certain stimulus either internal or external. Most automatic processes require a lot of training to develop fully, but once learned they are difficult to alter or suppress (Shiffrin and Schneider 1980, 25).

As regards the controlled processes the activation of a sequence of nodes requires the attention of the subject. It is probable that one can control, at a time, only one process unless two or more simultaneous processes are so slow that one can share the time and attention with them serially in a rapid alternation (Simon 1979, 4-5).

Both automatic and controlled processing demand retrieval in LTM. It is clear that retrieval is best when the conditions of encoding and retrieval are most similar. Of course we can retrieve some information also in the STM,

e.g. , to combine some pieces of information into a larger block or chunk. It is suprising how fast this retrieval can be done. In an experiment in which an examinee had to compare if a given number was one of those shown earlier (and located in STM) the rate of comparisons was one in 35 ms (thus thirty comparison in a second)(Sternberg 1969).

Besides knowledge structure and executive mechanisms for processing, the LTM consists also of learned ways to select, manipulate, and use information. These strategies have become very central in researching complex learning and performing processes like problem solving. It is the strategies and the increasingly conscious control and regulation of them that develop through a variety of problematic situations. That is the main reason why adults usually can recall more details than children in different life situations.

Gagné and White (1978, 195) have gathered research results and proposed explanations for them in order to get a general framework of different memory structures and corresponding performance outcomes.

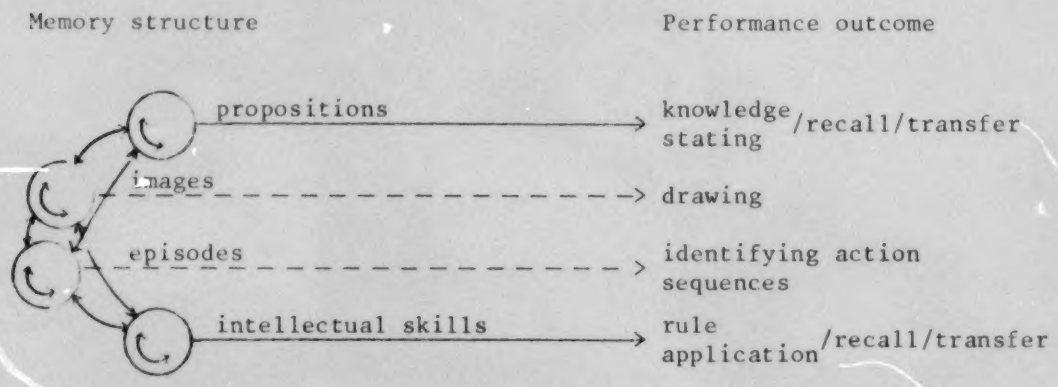


Figure 3.2.2 Memory structures and functioning (Gagné & White)

They distinguish four kinds of organized structures: 1) networks of propositions, 2) images, 3) episodes, and 4) intellectual skills. They tend to think that attempting to get fewer categories is not successful. The three categories of the model, namely proposional, imaginal (especially visual and auditory), and episodic ones, have been studied by a number of prominent cognitive scientists (e.g. Anderson and Bower 1972; Collins and Quillian 1969; Kintsh 1972, 1975; Paivio 1969, 1976; Rumelhart et al. 1972; Tulving 1972). It seems to have practical usefulness to know that memories established by learning may really have these different forms, often several

forms together. Propositional memory may have the primary position but the other forms are an aid to learning, probably depending on the individual and situations to varying degrees. Several possibilities to make this useful in school practice are suggested in detail by Gagné and White. These are concerned with teacher questions, processing of images and episodes, and choosing instructional materials. The suggestions are, of course, innovational and tentative. Including the fourth category of intellectual skills makes the system very useful, but its nature is obviously quite different from that of the others. Intellectual skills, the know-how, may be connected in memory to the contents of the other categories, especially to propositions. Strategies may be included in this category or they may form an additional category of skills.

4. Tentative Combining of Processes and Aptitudes

In order to specify information-processing abilities of different individuals two methodological approaches have been used. One is based on correlations between different aptitudes (measured by psychometric tests) and information-processing abilities. This approach can be called correlational approach. The other is based on identifying the process components or task performances straight-away. This approach is called component approach. The former can be characterized by the question "What does it mean to be high verbal?" and the latter by "What do psychometric tests test?" (Pellegrino and Glaser 1979). The answers are sought in terms of information-processing abilities.

One of the first really interesting researches connecting psychometric testing and cognitive processing was that of Hunt et al. (1973). The subjects were university students for whom the indices of verbal ability and quantitative ability were available and who were invited to participate in laboratory experiments. In the experiments several processing performances were measured: a) naming time when the two letters shown were the same (Aa, Bb) or different (Ab, aB), b) ability to retain the order of the letters, and c) checking time to respond when the number shown was or was not among the numbers previously shown and located in STM. The conclusion the researchers drew from results was that the high-verbal subjects (above the median of verbal testing) are

a) faster in accessing information in LTM, b) better in retaining the order of information in STM, and c) faster in manipulating information in STM than the low-verbal subjects (below the median). The quantitative ability was related to the ability to resist interferring information in STM. In general, the researchers claimed that verbal ability is related to the speed with which an individual can manipulate information in STM and that the quantitative ability is related to resistance of interference in STM. The small number and the selection of the subjects are limitations of the research.

The replication study by Chiang and Atkinson (1976) failed to find a significant relationship between verbal ability and STM processing speed; the data for males seemed to support Hunt´s results while that for females did not. In studies concerning different age groups the developmental aspect further complicates the picture. Keating and Bobbit (1977) observed that a group, mentally above the average as measured by Raven´s progressive matrices, performed faster in the two tests (a) and (c) (mentioned above) than the average group but high ability group had achieved the near-adult level of functioning as early as the age of 9; the average group, instead, developed in functioning and achieved the adult level by the age of 17. The phenomenon becomes even more complicated, if in test (a) pictures are used instead of letters.

There are, of course, still other properties in information-processing than speed, which is the easiest to investigate. Individual differences may be due to the holding capacity of working memory, the type of code used, the scope of attention-demanding and automatic processes, pattern matching in LTM (knowledge schemata), and mnemonic and executing strategies. In fact, in all these respects great differences have been observed between individuals, but these variables are difficult to define and investigate; as yet very little generalizable and consistent research information can be registered concerning psychometric test abilities. It might be more reasonable first to focus attention to intellectual activity itself in terms of cognitive components and theories and try to distinguish consequent components in this activity.

4.1 Cognitive Components

There is no prominent, comprehensive theory of the basic processes of human performances. What we have now is a number of theories suggesting what really happens in performance processes. The means by which the processes are investigated are different: psychological laboratory experiments, computer simulations, and neuropsychological methods.

In order to outline the processes, an attempt is made to organize the different levels and functions into a hierarchy. One typical hierarchy is the following (e.g. Lawson 1980, 147):

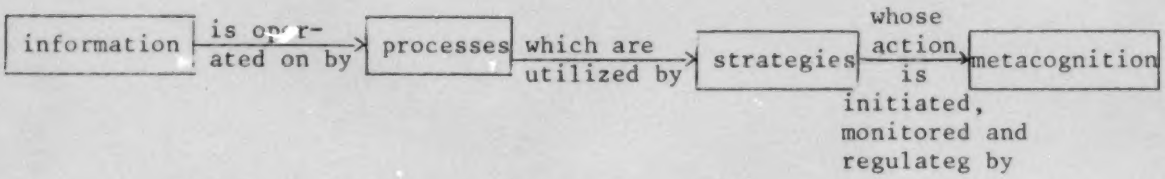


Figure 4.1.1 The hierarchy of cognitive processes

In this model the processes are concerned with input information and are different from one another like perceiving, encoding, searching, retrieving, manipulating, and some operations in STM like chunking and rotating. The processes may then be utilized in different ways according to the relevant strategies developed by the individual who is confronted by certain tasks. The term strategy originates from Bruner et al. (1954) and means the ways of selecting, manipulating, managing, and outputting information. Metacognition controls and makes decisions about the development of the strategies. Consequently, this stage consists of planning and decision-making. The model does not include the attention and arousal stage which regulates all processing.

In the following we describe some processes, which have been intensively researched, especially from the viewpoint of individual differences and instruction.

Memory processes. One primary ability in Thurstone's factors was memory which meant immediate memory ability. Even earlier, memory has been included in the concept of intelligence ever since Binet's time and great individual differences have been found. Cognitive theory introduced the concepts of memory span, a limitation of STM capacity, and memory processes such as

encoding and retrieval.

In order to enlarge immediate memory capacity the human being can group or chunk information elements into larger recognizable units (chunks) by means of strategies. The duration of immediate memory can be increased by rehearsal which is thus one memory strategy. Instead, there is no clear evidence to support the fact that memory span itself could change with age or be changed by practice (Chi 1976). Memory capacity varies within an individual in terms of the types of material and the age of the individuals using the same material. This is not, however, an evidence of a larger span because chunking or organizing strategies change; an adult remembers a greater number of separate letters and other objects than a child because he has more words, number series, experiences, and special tricks in LTM.

Can mnemonic strategies, such as rehearsal, chunking and organizing, be trained ? In a number of researches one or several simple strategies have been taught. For instance, it has been shown that a retarded person can acquire rehearsal strategies and improve his memory performance (Butterfield et al. 1973). Attempts to influence individual differences have, however, failed. Children have not been able to generalize the use of acquired strategies. That is why researchers have started to consider the question of mnemonic strategies from a higher viewpoint,in terms of metamemory (Kreutzer et al. 1975). Of course, strategies should be available, but this is only a necessary condition. What is also needed is an analysis of task demands, strategy selection and use, and often flexibility to modify, change, and combine strategies. The main problem in teaching is one of co-ordination and control of mnemonic strategies and that is the scope of metamemory (see Brown and DeLoache 1978; Chi 1978).

Spatial rotation. One basic mental process, the functioning of which is quite well-known, is mental rotation, i.e. the rotation of visual codes in STM.In the experiments the subject has to decide if the figure shown in different rotated positions is the same (normal) or mirror image of the initial figure. This experiment is similar to some psychometric tests of spatial ability but now the reaction time is measured as a function of angular separation from the initial (upright) position.

The results were interesting: when the figure was rotated from 0^{o} to 180^{o} the reaction time increased rather linearly and then from 180^{o} to 360^{o} it decreased just as regularly. The conclusion was that the subject really

rotates the figure mentally clockwise or counterclockwise whichever rotation direction is closer to the initial position. Similar results were obtained in three dimensional rotation but in this case the reaction times were longer (Cooper and Shepard 1973; Shepard and Metzler 1971).

Though the main results of mental rotations are similar in two and three dimensional spaces, there are also significant differences. In a three dimensional space the process is not just rotation but a sequence of subprocesses (Carpenter and Just 1978). By means of eye movement investigation it was possible to decompose the total process into several stages: first the initial searching process to identify the important parts of the initial figure and the rotated figure, then the mental rotation of one part of the figure, and at last checking rotation of the remaining parts of the figure. Because of the complexity of the process the individual differences were great concerning the different sources, such as organizing, sequencing, and executing the subprocesses. It means that the strategies and metacognition are relevant and important when one is dealing with space figures. It also explains why space geometry (in three dimensions) is much more difficult for some students in school than plane geometry.

Reasoning. The more complicated the mental processes the more difficult is the analysis of them into relevant subprocesses according to actual use. There is a number of researches of different reasoning processes but very few generalizable results. The main types of tasks generally used have been inductive tasks (letter, number, picture, and special geometric forms series), deductive tasks (syllogisms etc.), and larger problems (puzzle, mathematical, and anagram problems).

Typical inductive tasks (letter series) were used e.g. in the research of Kotowsky and Simon (1963, 1973, republished in Simon 1979). What they could report as results were individual differences in serial pattern (rule) generating, the order of difficulties to find relations, such as same (cdcdcd_), next (abmcdmefm_), backwards next (in alphabet: abyabxabwab_), and combining these (pononmnmlmlk_), and the time spent. The idea to make a distinction between the knowledge and routines available, and the executive that monitor and control the use of knowledge, was also reported. The idea is not new because to solve any problem the initial knowledge (terminology etc.) is necessary but not sufficient. What is still needed is the skill to

use knowledge and routines, i.e. strategies and metacognition.

Many researches also show that reasoning can be taught, i.e. students can be trained to improve their performance of similar tasks (e.g. Holzman et al. 1976). In the research mentioned the students from grades 1 to 6 were trained to discover the interletter relations and periodicity in letter series tasks. They learned to acquire a strategy which was needed in the tasks, even in difficult ones.

Inductive reasoning which is needed in a series of extrapolation tasks may be applied to a number of other types of tasks such as analogy tasks. From a more general point of view solving this kind of problems involves first the identification of the components of a task and then fitting them together in a pattern (Greeno 1978). How basic a process inductive reasoning is to information theorists is to be seen in the following quotation (Simon and Lea 1974, reprinted in Simon 1979): "(F)rom a logical standpoint the processes involved in problem solving are inductive, not deductive. (T)he proof of a theorem in a formal mathematical or logical system is a deductive object; that is to say, the theorem stands in a deductive relation to its premises. But the problem solving task is to discover this deduction, this proof; and to discover process, which is the problem solving process, is wholly inductive in nature." While saying this Simon and Lea are only partly right. In fact, the usual discovery process can be both inductive and deductive; the problem solver may discover one step (inductive) to advance and then draw a conclusion (deductive) and go on. There still remains the question of whether this kind of extension of the conception of induction is of any benefit when one is analysing problem solving. In letter series tasks there are questions concerning a very pure type of inductive reasoning, but in complex problems this kind of reasoning skill is not sufficient (see Leino 1977).

Analogical reasoning has been studied since the early decades of our century. Tests have been either verbal or nonverbal. The underlying processes have recently been studied by Sternberg (1977). The analysis shows that there is no single process to cover analogical reasoning. (Sterberg's theory will be considered in detail later on.) It is also obvious that verbal and nonverbal tasks demand at least partly different processes for even the first subprocesses of encoding differs greatly in verbal and nonverbal tasks; verbal information tends to be coded successively and visual simultaneously (nonverbal analogy tasks are usually visual)(Luria 1966a,b; Das et al. 1979, 49). The

Difficulty of a problem depends on the number of operations or attributes that must be represented in STM in order to draw an inference concerning a given case. Memory capacity can easily be extended and in this situation some strategy is needed. This can be e.g. working backward strategy which helps to divide the problem into relevant parts. In verbal analogy tasks the semantic network in memory is very important (see Pellegrino and Glaser 1979, 207-211).

Problem solving involves reasoning; from the initial stage of a problem, one has to reach the goal state and this happens through a sequence of intervening states connected with each other by logical moves or operations (see Wickelgren 1974, 10-20). To discover a sequence from beginning to end is to solve a problem. The operations from one state to another are subprocesses of problem solving and they are to be mastered, but the main difficulty lies in selecting and ordering these subprocesses, which is the scope of strategies and metacognition. It is probable that the selection of the subprocesses is essentially based on analogical reasoning (see Pask 1978, 207).

Whenever a problem solver achieves a solution he learns also the general pattern used in the process. That is why many researchers consider problem-solving to be closely connected or even identified with a type of learning (e.g. Gagné 1977; Pask 1978).

Strategies. We have distinguished the mental opetations which concern information and which we have called (mental) processes from those concerned with mental processes; the latter are called strategies. Strategies are by their very nature controlling processes. In the following we classify strategies in terms of cognitive processes: 1) attending and perceiving processes (perceiving strategies), 2) coding for temporary or permanent storing (combining and organizing strategies), 3) retrieval from LTM (remembering strategies), and 4) manipulation in LTM (thinking strategies) (see Gagné 1977, 167-179).

The first group of strategies control perception, i.e. on what kind or type of objects perception processes are focussed. Individuals differ to a great extent with regard to the kind of information they see or seek in certain situations. For instance in Witkin's Embedded Figures Test the perceiving strategies play an important role though the cognitive style of field-dependence/independence is connected with other processing stages as well. Perceiv-

ing strategies can be changed and taught. The teacher can direct the attention of students to different features of a picture, speech or text and thus develop certain strategies to attend and perceive.

Coding strategies refer to different ways to group information in STM and organize it into LTM. We have already briefly discussed simple coding strategies while dealing with memory processes. When dealing with more complicated strategies, e.g., studying conjunctive concept for rule forming over given cases some researches need to be mentioned. Bruner with his colleagues (Bruner et al. 1956) observed that in order to find a rule concerning cards with different figures different strategies were used. Depending on research method the strategies could be classified into reception (wholist, partist) and selection (simultaneous/successive scanning, conservating focussing, focus gambling) strategies. The attributes were very clearly identifiable (shape, size, color). In this experiment different combinations were coded in STM for rule forming. Waters (1980) varied also the attributes in terms of their clarity for identication. The other conditions were similar to those in the research of Bruner et al. The results showed that when the basic attributes were difficult to perceive the experiment concerned not only coding but also perceiving strategies.

Encoding information into LTM can be aided by many mnemonic strategies. One is organizing information according to some semantic code such as common attribute (all objects are fruit, quadrangles, professions, or same material etc.), another is elaborating information according to some common referent such as different topics, familiar or funny sentences etc. connecting the objects to be remembered. Of course, individuals use many ways to form an attribute in common of a relationship between the objects to be remembered. Pask and Scott (1972) distinguished two groups, serialists who formed a relatively simple relation (usually pairwise) to connect objects, and holists who formed a complex relation or superconcept.

Retrieval as well as encoding strategies vary in complexity. If one wants to retrieve some events of the previous day a good strategy is to reconstruct the day's events step-by-step (episodic memory). Many speakers mentally locate the successive parts of a speech in various places in a familiar house (the introduction in the front hall etc.). At the time the speech is to be given the speaker just takes a mental walk through the house. This strategy of the "method of loci" is a good example of retrieval tricks.

The efficiency of different strategies as a function of age has been studied by Kobasigawa (1974). His research shows that the efficiency of one strategy may depend on the student's age and that of another be equally good at different ages. The teacher can manipulate retrieval processes of students by different cues, such as using key words, telling a structural story about the successive stages of a task to be remembered, or simply by asking students to pay attention to the inportant features of a task.

The fourth strategy group, thinking strategies have usually been considered in close connection with problem solving. In spite of the fact that to teach problem-solving skills is rather difficult and not well-known, there are some teaching programs that have proved to be efficient (Bloom and Broder 1950; Rubinstein 1975), some guide books for systematic problem-solving (Polya 1957, 1962, 1965; Wickelgren 1974), and also some researches about problem-solving strategies (see Tuma and Reif 1980). Maybe most of the impulses for systematic investigations of problem-solving mechanisms have come from computer simulations of human intelligence (Newell and Simon 1972; Simon 1979). However, the problems have been rather artificial puzzles and games, the nature of which is quite different from real-world problems.

Within the last five years especially in mathematics the problem-solving processes have been intensively researched (see Goldwin and McClintock 1980). There heve been two different methodological approaches: one is a very detailed catalogue of all possible sequences of student performance, the purpose of which has been to describe precisely the actual problem-solving processes, and the other is the investigation of some broad patterns of strategies in problem-solving processes for the purpose of getting useful information to be used in teaching. While looking at the mess (erratic performances) resulting from the former approach one cannot help preferring the latter approach; prescriptive norms are often quite useful in attemps at describing some vague phenomenon. (This theme has lately been analysed by Schriven 1980).

Some general strategies for solving problems have been proposed by Polya (1957, 1962, 1965) - Polya used the term heuristic to refer to the study of the methods and rules of discovery and invention - and sophisticated by Wickelgren (1974): 1) drawing inferences (making transformations of the goals of the problem or the givens), 2) classifying action sequences (a developed stage of trial-error method), 3) hill-climbing (systematic rolling out all possibilities from the initial to the final state and picking a favorable

path), 4) subgoals (breaking up a problem into subproblems and solving them), 5) contradiction (indirect proof, multiple choice-checking etc.), 6) working backward (starting from goal and trying to find preceding state etc.), and 7) special cases - generalization.

Developing this kind of thinking strategies has to be done in terms of favorable conditions (Gagné and Briggs 1979, 72). The students need to be given different problem situations with new ways of solution reguired, and a variety of available strategies from which to make a choice. Unfortunately, there are very few researches to indicate how and at what age a certain strategy is best taught.

Metacognition. The term metacognition refers to one´s knowledge, selection, use, monitoring, and regulating of one´s cognitive strategies and processes (Flavell 1977; Flawson 1980). The awareness of one´s skills is often a necessary condition to use them; if a student notices some of his classmates using some skills taught, he believes that he can also do it himself (Skemp 1979, 87). The students appear to be more sensitive than younger children about the usefulness of some strategies.

There is not much research about metacognitions. Two comprehensive reviews of research are worthy of mentioning: Brown 1977 and Flavell & Wellman 1977. Most of researches concern training the metacognitions of mentally retarded children or the awareness of some memory strategies of young students. One aspect is well-known: the deficiencies in complex situations are not necessarily an indication of the absence of relevant strategies or processes; the student might simply be incapable of recalling these or even trying to do that.

4.2 Sternberg´s theory

The purpose of componential analysis is to identify the components of mental operations that underlie a set of related information-processing tasks, and to discover the organization of these component operations in terms of their relationships both to each other and to higher-order constellations of mental

abilities (Sternberg 1977, 65). A component is an elementary information process that operates upon internal representations of objects or symbols, transforms a sensory registration into conceptual form or one conceptual representation into another,or transforms a conceptual representation to a motor activity.

Problem-solving in a very wide sense is the most important form of human thinking. On one hand, analogical reasoning can be regarded as the most important form of problem-solving. Some researchers say that all problem-solving can be expressed in analogical form (e.g. Pask 1978, 123). While attempting to form a general model for performances on a large variety of analogical tasks Sternberg (1977) has actually formed a rather comprehensive theory of human thinking. Because he also verified many parts of his theory, we will discuss this theory briefly as an example of cognitive component theories.

Sternberg proposed five mandatory components and an optional one to compose the elements of his theory. The components are of three types: A. attribute identification, B. attribute comparison, and C. control. In addition to these components the theory consists also of a specification of the combination rule for the components.

The six component processes include: A1. encoding the terms of the analogy problems; B1. inferring the rule relating the first term of the analogy to the second term (the outcome is stored in STM); B2. mapping by a higher-oder rule the relationship between the first and third terms; B3. applying the results of the inference and mapping processes to the third term to generate a proposal of a correct answer and to evaluate it; B4. an optional justification process which is used to select one of several alternative answers (not strictly correct ones); C1. a control process by which the subject prepares for solving an analogy problem, monitors the solution process, and transforms the solution into a response. An example is given to clarify these concepts. When a driver comes to a crossing at the moment of red lights (encoding term: red light) the inference is the discovery of the relation between the red light and stopping the car (the second term). The third term in this example is the green light and mapping is the discovery of the relation between red and green lights. Application process means the connection between the green light ant the permission to drive on.

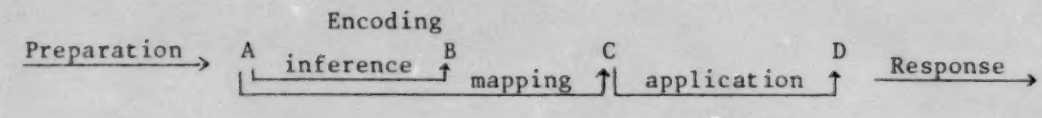


Figure 4.2 Schematic representation of analogical reasoning theory (Sternberg)

The combination rule is simply additive: solution time is the sum of times spent on consequent component processes. Solution difficulty is correspondingly the sum of component difficulties. While testing his theory Sternberg distinguished between models with all or only some of the component processes and used several materials (verbal, pistorial, and geometric).

Using latency measures for different component processes Sternberg related these to general reasoning scores and obtained multiple correlations ranging between .69 and .87. Processing speed of different processes did not correlate with general reasoning in the same direction for all subjects. Thus the speed does not seem to be an adequate measure for global abilities; some subjects need more time for reasoning than is expected on the basis of the times they need for component processes. The same result has been obtained by other researchers while trying to describe global ability by means of component abilities (e.g. Krutetskii 1976; Leino 1978).

Sternberg also attempted to combine classical group factors (abilities) and his own theory (1977, 318). He regards group factors to be somewhat arbitrary compared with the component processes, because the former are based on inter-individual differences and the latter on intraindividual ones. However, this does not necessarily mean that the factors would not describe intraindividual differences as well.

4.3 Carroll's Approach

Another example of attempting to explain aptitudes, i.e., traditional mental abilities, in terms of cognitive processes is the analysis by Carroll (1976). This analysis shows very clearly how complex it is to match task characteristics, in all their width, to cognitive processes, in all their depth. Though Carroll's

analysis is based on subjective intuition, it may lead to an important research orientation for generating a more valid model of intelligence than Guilford's SI model.

Carroll took a representative sample of cognitive psychometric tests assembled by French, Ekstrom and Price and started to split all items into atomistic requirements for performance in terms of cognitive processes. For this purpose he first developed a uniform system for coding the characteristics of the tasks represented by the items of each test. The coding scheme consisted of six main categories: I stimulus materials, II overt response to be made at the end of task, III task structure, IV operations and strategies, V temporal aspects of the operation or strategy, and VI memory store involved. There were more than a hundred subcategories. After generating these Carroll coded all items of 48 tests into this system.

This study resulted in cognitive processes identified as being characterictic of each of the 24 group factors (abilities). (French et al. had reported these tests consisting of 24 primary factors.) While appraising the final table of all characterictics it has to be remembered that the main purpose of the study was innovative and methodological. What was accomplished was a tentative proposal as to the nature of aptitudes. Hence the results reported are scarce. For example, verbal ability (V) is almost exclusively dependent upon the contents of the lexicosemantic LTM store, no strategies are considered important; the spatial ability (S) involve the capacity and rotation rate of STM, no strategies are seen to be needed. If general reasoning (R) and deduction (D) (syllogistic reasoning Rs) are compared in the table reported, hardly any differences are noticed. Even these examples show that Carroll's subjective interpretations are deficient; the precision and specification should be essentially increased if this methodology is going to have greater significance.

5. Cognitive Styles

In the study of individual differences in terms of cognitive processes a useful distinction can be made between the content of cognition and the manner

in which the behavior occurs. The former concerns the question of what, i.e. what is the information to be processed, what processes are used and how successful processing is. In the latter the characteristic modes of processing are dealt with in terms of typical performances. It is the latter aspect of cognitions that is the scope of the concept of cognitive styles.

Cognitive styles can be conceived of as preferred or habitual ways of processing information. They are not just simple habits in the technical sense of the word but deep-rooted, pervasive, and consistent ways of perceiving, remembering, thinking, and judging (see Kagan 1976, 1). Because different experimental situations may favor different styles, i.e. one style may be more successful in certain situations than another, the styles are easily involved with abilities. However, abilities are value directional: having more of an ability is better than having less. Cognitive styles are,in contrast, differentiated: each pole of a style has adaptive value in different situations (Messick 1976, 9). An ability is also more limited in scope than a style.

Cognitive styles develop slowly along with experience and are very resistant to attempts at modifying them. Styles also come to view in different levels of cognitive operations. Hence, it might be reasonable to distinguish styles and strategies (see Messick 1976, 6). Another argument in favor of doing so is the fact that a style is more global extending over a wide variety of situations than a strategy; a style can be considered to cross both processes and strategies. - However, the terminology is not well-established.

Some cognitive styles are closely combined with temperamental traits, e.g. the dimension of reflectiveness versus impulsiveness. Because a cognitive style, at either extreme in particular, means a strong predisposition to prefer some types of situations, the styles also penetrate motivational and affective factors. Thus Messick (1976, 10) regards them,in terms of factor-analysis, as higher order factors which link first- or second-order factors, such as abilities, temperamental traits, and affective consistencies. Hence, cognitive style can be regarded as an individual's latent perception and response dispositions through a wide variety of tasks.

Some researchers regard the term learning style as a synonym for cognitive style. Because cognitive styles affect learning process as well as other processes, e.g. perception process, they are not considered to be synonyms in this research. (See also Leino, A-L 1980.)

When cognitive styles are measured, the subjects have been given e.g. a set of picture cards to group and give reasons for grouping them the way they did. The responses have been scored and dichotomized by using the median of the scores. This procedure has given two types of subjects which are rather different in average but there are also subjects taken from each group with very small differences in the style in question.

5.1 Some Dimensions of Cognitive Styles

There is a large amount of cognitive style researches and attempts at finding new style dimensions. Only few dimensions have survived and proved to be of importance in some respects.

Field-dependency. The most thoroughly studied cognitive style is in dichotomized form field-dependence versus field-independence. Individuals who are unable to free themselves from the visual stimulations, such as a tilted frame in respect to the uprightness of a rod, are also unable to free themselves from other irrelevances. A field-dependent person cannot articulate a hidden figure in embedding context or a simple tune in a complexity of sounds, but tends to experience events globally in an undifferentiated fashion (Witkin et al. 1954; Goodenough 1976; Witkin et al. 1977). A relatively field-independent person, in contrast, can keep items separate from the surrounding field, can restructure the organization of the relatively unstructured field, in short, is competent in analytical functioning in materialized situations.

This stylistic orientation is very consistent and far-reaching. Field-dependent persons prefer social contacts and are more able in social skills compared with field-independent persons who are more impersonally oriented. In situations involving learning and remembering, field-dependent persons pay more attention to social cues, acquire more information from the social environment and are more sensitive to external feedback and criticism than field-independent ones who perceive more in the material environment, acquire generalizations more easily and learn more under conditions of intrinsic motivation. In promlem-solving tasks a field-dependent student needs more explicit instruction of strategies and more exact definition of performance outcomes than his field-

independent class mates. Field-independent students are more autonomous and active to participate in the teaching process than field-dependent ones who tend to be spectators (Witkin et al. 1977). Both types acquire equal amounts of knowledge but of different nature.

Because field-independent persons can more easily than field-dependent ones perceive some critical elements in a diffuse context, they have appeared to be better problem-solvers and higher in intelligence. That is why some researchers consider field-independence to be an ability or closely connected with abilities (Horn 1976; Guilford 1979, 135-136; Kogan 1976, 5). As a matter of fact, field-independence indicated by Embedded Figure Tests (EFT) correlates highly with general visualization ability (Vernon 1972). Other measuring instruments of field independence, such as rod-and-frame (RFT) and tilted-room-and-chair (TRC) give an additional component of visuokinesthetic functioning. All these measures are typical quantifications of abilities and evidences of field independence being closely involved in certain kinds of situations, but they do not tell anything about the subjects' preferences and response modes. Field-independent subjects are better problem-solvers in problems with some critical elements hidden in diffuse contexts and higher in intelligence because intelligence tests include numerous items which demand analytical functioning. In social problem situations field-dependent subjects are superior, these problems being very important in ordinary life but lacking in intelligence tests.

It might be of importance to mention that the trait of field dependence starts to develop at the age of 4-5 years and it has tendency to differentiate with age (Kogan 1976). There are some references in literature explaining field dependence in terms of cognitive processes (Das et al. 1979, 140-141; Vasgird 1980) by connecting simultaneous and successive processing to field independence - field dependence but no empirical attempts have been made in this respect.

Levelling - sharpening. One of the first examples of cognitive style studies was that of Holzman and Klein (Holzman and Klein 1954, Klein 1958). They gave subjects to estimate squares of differing sizes, presented in random order in each set. In the first set, the sides of squares varied from 2 to 7 (inches), in the second set from 3 to 8, and finally from 9 to 14. Levellers did not notice the gradual shift, they based their estimates on relative sizes. At the beginning a 6-inch square was considered large but at the end even a 10-inch square was regarded as small. Sharpeners appeared to keep the absolute

basis in mind for the estimates.

This style has been noticed to be consistent in various situations: loudness of tones, heaviness of weights etc. Levellers tend to be more sensitive to similarities and sharpeners to differences. Levellers tend to consider each successive object exactly the same as the previous one so that the new object seems to lose its identity. Sharpeners, in contrast, tend to seek differences and contrastive points in successive objects whether these are particularly relevant or not. In terms of cognitive theory this style dimension concerns especially the way to encode information into the LTM and to retrieve it from there.

Focusing - scanning. Closely related to the dimension of levelling - sharpening is the focusing - scanning continuum. Scanners attend broadly to the total perceptual field but intensively in a sequential fashion. It is not easy for them to ignore background irrelevances. Focusers, in contrast, seem to notice the surrounding world in a narrowed, discriminating way. They find features and relevances and easily ignore irrelevances (Schlesinger 1954; Gardner and Long 1962a,b). They seem to accept their perceptual experiences in a more relaxed and less critical way than scanners who are often cautious and distrustful. Scanners are usually sharpeners, but not all sharpeners are scanners (Moursund 1976, 277). The focusing-scanning style concerns the way attention is employed.

Reflection - impulsivity. Another well-studied cognitive style suggested by Kagan et al. (1964) is the dimension reflection - impulsivity. While studying young children's so-called analytical or relational pairing of objects Kagan noticed that analyzing usually took more time than the other response type. He started to study the response process in terms of response time and response errors.

Impulsiveness is usually measured by the matching familiar figure test (MFF). Impulsive persons are those below the median in response time and below the median in numbers of errors, whereas reflective persons are those above the medians. The person who is either slow-inaccurate or fast-accurate has usually been omitted from further considerations. Of course, this causes methodological troubles in the analysis of variance, the method most usually employed in these studies. Another comment is that the original relationship between analytic

responses and reflectivity has failed to be found in further replications (Messer 1976, 1033).

The studies of reflection - impulsivity have shown that the response time component is relatively independent of general abilities but the error component is moderately related. Reflective persons tend to gather more information more carefully and systematically than do impulsive ones. Relatively extreme impulsivity tends to indicate reading difficulties and other learning disabilities. Children typically become more reflective with age when they find the usefulness of delay. It has also been found that impulsivity can be modified by means of appropriate training. (See Messer 1976).

Guilford suggests that the reflectivity dimension might be the reverse to the temperament trait of thoughtfulness or identifiable with the second-order factor of introversion - extraversion. However, attempts at matching reflection - impulsivity to personality or social variables have not been very successful.

If problems contain response uncertainty the impulsive students tend to use trial-and-error method in problem-solving situations. Reflective children tend to use more mature strategies and hence process more efficiently (McKinney 1975).

Conceptual categorization. Many studies have shown that individuals have strong spontaneous tendencies to organize atomistic pieces of information into categories and subcategories and use imposed grouping principles to aid recall. Category clustering is neither just an experimentally induced phenomenon nor just a strategy adopted in artificial situations of laboratory testing (Cohen 1977, 9). Because any material can be classified by using many schemes, different subjects can use different but intraindividually consistent grouping strategies in certain situations. The general mode of grouping objects in a consistent way in a variety of different situations can be called a conceptualizing style.

In an early work by Kagan et al. (1963) three different conceptualizing styles were identified: 1) analytic-descriptive, where objects were grouped on the basis of some concrete similarity, either of wholes or of parts (e.g. "they all have stripes"), 2) inferential-categorical, where the basis for grouping was not an obvious attribute of the objects but an inferred one (e.g. "they

all are professional men"), and 3) relational-contextual, where objects were grouped on the basis of some relation between the objects (e.g. "mother and her baby"). These styles appeared to develop in a certain mental order, with the relational style being more characteristic of young children, the analytic style of older children and widely used by adults, and the inferential style being especially typical of adults. This general tendency does not explain the whole phenomenon (see Kagan et al. 1963; Sigel et al. 1967).

Conceptual complexity. In the experiments mentioned above a subject got a picture of an object which was to be paired with one of three other pictures. If a list of items of information was given and a subject was told to classify them, another aspect came to view: different subjects used different numbers of categories. The number is thought to indicate the complexity of conceptualization. A small number indicates a simple conceptual system, usually a concrete one. A large number tends to be connected with a complex conceptual system composed of many distinct dimensions or concepts. This idea has been developed by Harvey et al. (1961) and further by Hunt (1971) into an instructional matching system, where the conceptual level of complexity of a student is matched to corresponding instructional environment.

Breadth of categorization. An experimental procedure where a subject is asked to give the extreme values of some item (e.g. annual rainfall, length of whales etc.), leads to an additional aspect of categorization, namely the breadth of categorization (Pettigrew 1958). This style reflects differential tolerance for error types. The narrow categorizer is thought to be conceptually conservative as opposed to the broad categorizer, who is more tolerant of deviant instances (Messick 1976, 15).

Serialism - holism. Pask and Scott (1972) have distinguished an addional dimension in the way to group objects. They found, on one hand, that there are persons who use simple and surface relations in connecting objects and, on the other, that there are persons who use high-level super-relations. The former are called serialists, the latter holists. We have mentioned this dimension as a coding strategy but it might be seen as a far-reaching style, which comes to view in various situations in which individuals seek and process information. Pask and Scott also found that if instructional material is written especially for either a serialist or a holist the other type of person is unsuccessful in using the material.

Das et al. (1979, 142) suspect that holists are used to processing information simultaneously whereas serialists tend to process information successively. They have tried to adapt their theory of simultaneous and successive cognitive processes to fit the researches and theory by Luria, a well-known neuropsychological investigator (Luria 1973, 1976). There is, however, no empirical evidence available.

5.2 Cognitive Styles in Mental Structure

There are very few attempts to allocate cognitive styles in comprehensive mental structure or to find out the structure of cognitive styles. One suggestion has been made by Royce (1973, 305-407).

Royce considered cognitive styles as higher-order constructs and compared them with values. (He used the term style factors and evaluative factors.) Both styles and values are embedded throughout the affective and cognitive structures. Royce delineated only one example of a possible hierarchy of styles (Fi. 5.2), because he felt it premature to spell out a comprehensive style structure at that time.

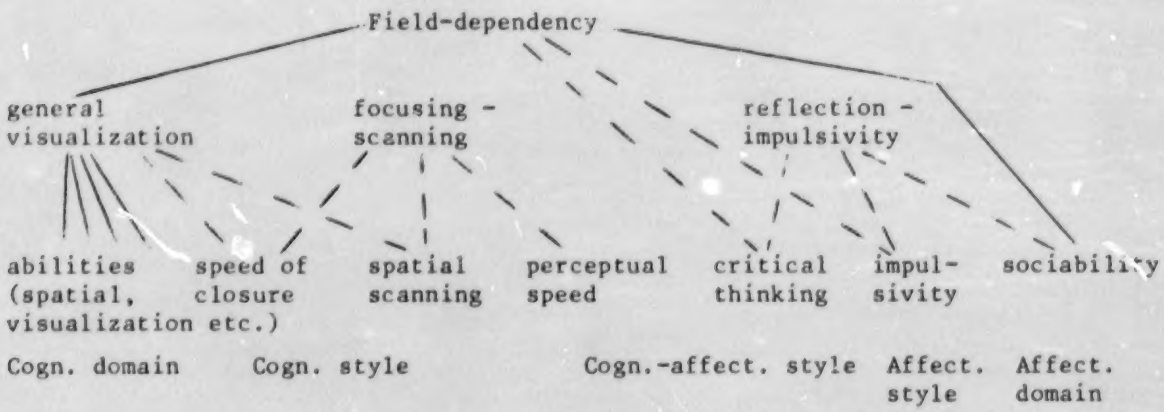


Figure 5.2 Some linkages of abilities and styles in cognitive and affective structure (Royce)

Royce distinguished cognitive, affective, and cognitive-affective styles on the basis of primary linkage. The example is partly empirical, partly rational-intuitive guessing.

The different levels of the figure refer to the order of factoring. For instance critical thinking is a first-order affective factor, the content of which is reflective, meditating preference of thinking whereas the speed of closure is an ability to comprehend quickly a perceptual field as a whole with the completion of unseen portions. Second-order factors are general visualization (ability), focusing-scanning (style), reflection-impulsivity (style), and also field-dependence, though the latter might be of third order. Royce regarded reflection-impulsivity as an affective style and field-dependence as a cognitive-affective one. Field-dependence is especially a perceptual tarher than conceptual style.

Conceptual styles such as categorization, complexity, breadth of categorization, and serialism-holism form another group of styles which connect perception modes and LTM closely together. This group is cognitive and conceptual rather than perceptual.

The dimension levelling-sharpening can be located between the group above and the focusing-scanning factor. Because of the lack of empirical evidence, the structure of these styles is not presented here.

Another way to organize cognitive styles is to consider them on the levels of cognitive operations. In this classification the conceptual styles deal with thinking processes, grouping and organizing strategies, and perception-memory -subsuming. Field-dependence emphasizes perception processes though it might be much more far-reaching in the direction of strategies and metacognitions. Reflection-impulsivity connects executing processes and a temperamental trait together.

6. Empirical Approach

In this report an attempt is made to discover what the theoretical framework concerning the relationships of the psychometric tests and cognitive processes could mean in terms of school education. In order to achieve the relevant level for the purposes of the study we have to use some context in which to operationalize the framework. Such a context is offered by school mathematics.

The developmental course of the research of mental abilities, described in the foregoing, can also be seen in the efforts to finding psychological basis for mathematics teaching. By means of factor analysis the basic dimensions of mathematical performances, such as they have appeared in the usual achievement tasks of school mathematics, could have been identified. The "primary" mathematical abilities, identified in this way, consist of numerical ability (N), spatial or visual abilities (S, Vz), reasoning abilities (general reasoning R, inductive reasoning I, and deductive reasoning D), and verbal ability (V) (see Thurstone 1938, Werdelin 1959, Treumann 1974, Leino 1977). After the stage of ability identification, researchers have developed psychometric tests which could have been regarded as pure instruments of each ability.

In addition to the "primary" abilities a general academic success factor and several specific factors such as a factor of arithmetic, geometry, or algebra have been found in individual investigations of the field. All these abilities have moderate correlations with each other and they could have been grouped on the level of second-order factor. Figure 6.1 schematically presents the structure of mathematical abilities (Treumann 1974, 201):

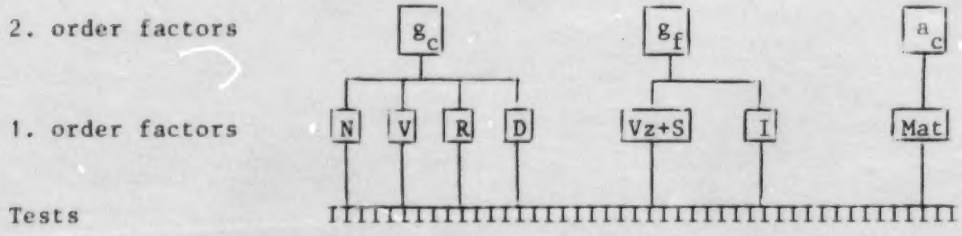


Figure 6.1. The structure of mathematical abilities (Treumann)

Numerical ability, verbal ability, general reasoning, and deductive reasoning have high loadings on the crystallized intelligence g_c and lower loadings on the fluid intelligence g_f whereas spatial and visual abilities, and inductive

reasoning are loaded more highly on fluid intelligence. The third factor a_c consists of general school success in mathematics.

The mathematical abilities, as such, have little effect on mathematics teaching because a teacher of mathematics needs also advice how to develop these abilities in school. In order to find out the relationships between abilities and mathematical activities in process terms some research attempts have been made in this direction.

Of all school subjects it is probably mathematics in the context of which most process researches have been made. This is understandable because problem-solving process is a corner stone in mathematics learning as well as teaching. For developing a theory of problem-solving instruction a number of basic studies have been made within the past twenty years. Only few are discussed here. (For a detailed review see e.g. Goldwin and McClintock 1960.)

Dunker (1945) used, in his early research, thinking aloud techniques to obtain data in order to determine in what way the students attained solutions of problems. The problems were quite difficult and demanded some either general or task-specific heuristics. Especially two heuristic processes were directly investigated: 1) analysis of the situation, and 2) analysis of the goal. Also the effects of heuristic hints were studied.

Kilpatrick (1967) improved the techniques used by Dunker for analyzing protocol material. In this technique the theory of Polya (1957, 1962, 1965) was adapted into a classification of heuristic problem-solving processes. However, many of the categories of Polya was eliminated because they were used by students hardly ever in actual solving processes. The final coding system was composed of: 1) preparation (drawing figure, changing condition (specification, generalization), performing exploratory manipulations), 2) recall (similar or related problem), 3) production (using successive approximation, questioning existence/uniqueness of solution, using trial-and-error/equation/deductive process), and 4) evaluation (checking solution). The major results were meagre: a) the best variable to distinguish the good and poor problem-solvers seemed to be using trial-and-error versus deductive method, and b) drawing a figure seemed to bear no relation to the mode or success of a solution (poor solvers drew poor figures, good solvers good figures but none of them used the figures to real advantage). It must be pointed out that only subjects of above-average could be used in the study. The number of the subjects was 56 and they were eight-graders.

Webb (1975) modified Kilpatrick´s coding system and made an attempt to determine relations between cognitive abilities, problem-solving abilities, and the use of problem-solving processes. The subjects were high-school volunteers taking secong-year algebra (N= 37 for analyses). The frequencies of the processes used by students were taken into consideration if the intercoder reliabilities were high enough and the total scores of the processes were given as the sum of three subscores (approach, plan, and result). Only 8 problems were used. In order to reduce the number of variables the principal component analyses were performed on three sets of tests: pretests (achievements, abilities, attitudes, and problem solving), heuristic strategies, and process-sequence variables. The new 4-5 components accounted for 60-70 % of the total variances in these three sets. Using regression analyses it was shown that a large overlap existed between pretest variables and heuristic strategies in accounting for the variance of problem-solving scores in the 8 problems mentioned above. Mathematics achievement and verbal reasoning were weighted on information retained,and pictorial representation was weighted on processes such as drawing and using figures.

Of the research[illegible], where Polya´s heuristic processes were first taught before investigating the actual processes used by students, those of Lucas (1972) and Kantowski (1975) can be mentioned. Lucas showed that the following processes were changed under heuristic instruction: a) the kind of notation, b) applying method/results of related problems, c) reasoning by analysis, and d) organizing data.

Kantowski taught geometric content using heuristic approach. In the pretest and post-test 8 volunteer students solved some problems by thinking aloud. In each problem a process-product score was given and the process-sequence was categorized according to a modified version of Kilpatrick´s system. The results showed that subjects given heuristic instruction increased their use of heuristic processes as the abilities developed, and success in problem-solving had a very good impact on the use of these heuristic processes.

Krutetskii (1976) used in his studies a large variety of different problems, usually rather small ones. Although he obviously controlled process variables - but did not specifically say so - his main purpose was to directly interpret the performances of the students as abilities. For instance 1) obtaining mathematical information was interpreted as an ability for formalized perception of mathematical material and for grasping the structure of a problem.

In the same way he analysed the other variable groups - 2) processing and 3) retaining mathematical information - into abilities. Unfortunately, he did not specify in which problems he scored each ability and how he did it. Thus Krutetskii´s investigations are not purely empirical but semi-clinical in nature. The way he selected the problem series is, however, most interesting. Also Krutetskii used particularly talented students in his studies, but he had, as a basis for comparison, the groups of average and incapable students, too. Using a large collection of tasks he made an attempt to characterize the mathematical abilities of students and, finally, the mathematical cast of mind.

The final stage of the developmental direction of mathematical problem-solving researches has consisted of attempts to discover a detailed characterization of problem-solving processes, possibly related to different types of tasks (see McClintock 1980; Kulm et al. 1981). The researches have shown that it is fruitless to describe individual processes in all details, because of enormous individual differences, but using rather large categories the main features of the differences can be caught in view.

6.1 Framework of the Empirical Study

The empirical part of the present report consists of the measurements of aptitudes and attitudes, and information-processing abilities and the analysis of the relationships between them. School achievements are an additional control variable. The design of the study is presented in Figure 6.1.

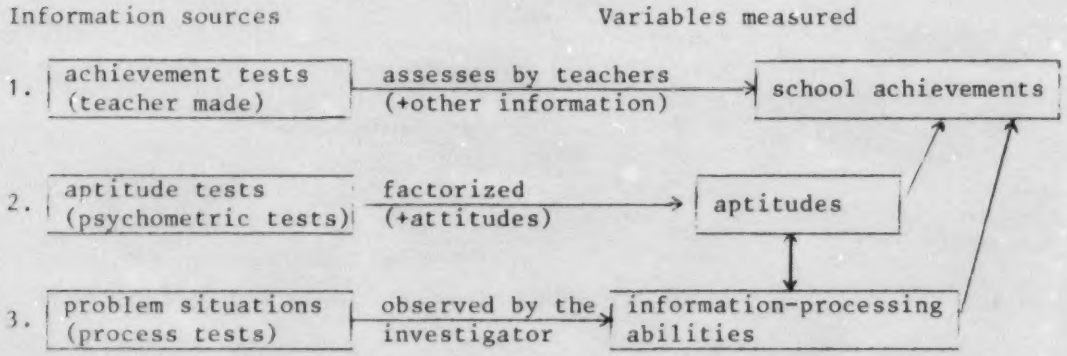


Figure 6.1 The design of the research

The aptitudes and attitudes are regarded as explaining variables of the assessments of school achievements in mathematics. The information-processing abilities are concerned to be more detailed and sophisticated variables than the aptitudes and can be used in explaining the achievements. The analysis of the relationships between the aptitudes and cognitive processes can be called a correlational approach, i.e. we ask what it means to be high or low in some aptitude variable in terms of cognitive processes.

The specific objective of this study is to derive from the analysis:

1) relationships between aptitudes (psychometric abilities) and information processes (processing abilities),
2) a characterization of different mathematical types or cognitive styles, and
3) a characterization of a high achiever and a low achiever as compared to an average one in terms of aptitudes, attitudes, and cognitive operations (processes, strategies, and metacognitions).

6.2 Subjects and Procedure

The first stage of the empirical study was the selection of the subjects, which was based on two main principles: first, the structure of the (mathematical) abilities and attitudes should be differentiated and relatively stable, and, secondly, the sample should be representative for the whole age group of students. The first point of view supported the choice of the age of 12-13 years or more (see e.g. Treumann 1974, 334-345), and the second one some grade of the comprehensive school. Thus three classes of seventh grade students were selected (N= 79). At the time of data collection some students were absent, which reduced the sample size to 67. Most of those who were absent represented the average group. The mean age of the students was 13.8 years. The number of girls was 34 and of boys 33.

Collecting the data concerning aptitudes and attitudes was a group test which took 5 hours per class.

In order to find out the thinking and performing processes of the students, the method was to interview students individually asking them to "think aloud" all cognitive and affective processes, strategies or heuristics, while dealing with the problems or behaving in the problem situations presented to them. If

a student could not answer or make any progress in the process, some helping questions were asked in order to identify the basic reason for the failure. If the student seemed to have forgotten some concept or fact necessary to progress, he was given the information and asked to continue. Interviews were tape recorded and all drawings or calculations made by students were also gathered. Because the process interviews took 2-4 hours per student, only 21 students out of the 67 mentioned above were selected for this part of the study. In this selection an attempt was made to get representatives from all types of students according the ability profiles.

6.3 Tests and Scoring

The aptitude and attitude tests were selected from Werdelin's researches (1958, 1961) because his tests were developed for the same age group as the one in this study. Werdelin is one of the most distinguished researchers in the field of mathematical abilities and attitudes. Only four tests with the highest loadings on each ability fartor were used (thus 4x5 = 20 tests). These classical mathematical ability factors were: numerical (N) and verbal (V) abilities, general reasoning (R) including also a test for inductive reasoning (I), deductive reasoning (D), and spatial ability (S) inluding also tests for visualization (Vz). These abilities are most commonly extracted by means of factorizing representative samples of mathematical tasks (see e.g. Treumann 1974). The tests selected from Werdelin's research (see Werdelin 1958, Appendix C) were:

Factor (in Werdelin's research)	Tests
N	Test No.30: Addition III; Test No.21: Addition I; Test No. 23: Multiplication; Test No.24: Equation(modified)
R	Test No.35: Plain Geometry(modified); Test No.29: Solid Geometry(modified); Test No.15: Arithmetic III(applied); Test No.16: Arithmetic IV(applied)
D	Test No.13: Syllogisms I; Test No.14: Reasoning; Test No.36: Syllogisms II; Test No.2: Number Series II
S	Test No.32: Lozenges; Test No.31: Figures; Test No.34: Form Boards II; Test No.26: Solid Figures
V	Test No.8: "Wrong Word"; Test No.12: Size Comparison (modified); Test No.10: Completion; Test No.6: Same-Opposite.

The attitude test modified by Malinen (1969), consisting of 27 items, was used with slight modifications (see Appendix 1).

The information processing tasks used in the study were selected in the way which could make it possible to discover most of the important characterictics of typical mathematical performances in different problem situations. The situations varied from simple symbol-operation tasks to difficult verbal and geometric problems. The subprocesses were classified into three main categories according to Krutetskii (1976, 350-351): IO) Obtaining mathematical information, IP) Processing mathematical information, and IM) Retaining and recalling (memories) mathematical information. Here, the first component comes to view when the problem is read or heard, and the second when a plan to solve the problem is conceived and carried out. The third component interacts with the first as well as the second. In the analysis of the three subrocesses the following variables were selected to cover the performances including verbalized thinking processes and strategies (see Krutetskii 1976, 350-351; Polya 1965, 36-53; Travers et al. 1977, 140-152; Wickelgren 1974, 10-46):

IO. Obtaining information:

IO1) perceiving the given information (facts, figures etc.)
IO2) fast perceiving geometric information in embedding context
IO3) finding out the relations between information given
IO4) grasping the formal structure of a problem

IP. Processing information

IP1) using trial-and-error method
IP2) using appropriate notations and combining them to the initial information (in particular, forming equation)
IP3) getting the expression of the solution
IP4) operating with numerals and other symbols
IP5) drawing inferences
IP6) generalizing objects, relations, and operations
IP7) changing the direction of reasoning (forward-backward)
IP8) curtailing the reasoning process or using some curtailing model
IP9) making helping drawing, figure, or grahp
IP10)processing fast (processing time/rate)

IM. Retaining and recalling information (memories)

IM1) recalling terminology, formulas, or concepts
IM2) recalling generalizations
IM3) recalling problem type (analogical problems).

To find out these processing abilities a collection of problem situations was selected for the test (Appendix 2). It consists of mathematical problem situations or tasks which partly resemble school problems and partly are different from them but, however, soluble by means of school mathematics. Most of the items of the process test are modified from Krutetskii´s tests (Krutetskii 1976, 100-175) consisting of arithmetic, algebraic, and geometric problems. The main categories of tasks are: 1) problems with unstated questions, 2) problems with incomplete information, 3) problems with surplus information,

4) problems with interpenetrating elements, 5) rule applying, 6) level of abstraction, 7) generalization of objects, 8) flexibility of thought, and 9) generalization of rules and operations.

In order to score the performances of the students certain items were used to indicate the existence of each variable:

Variable	Identified in Items
IO1	1.1-1.10, 4.1-4.5
IO2	4.1-4.5
IO3	1.2-1.9, 2.1-2.4, 8.2-8.4
IO4	1.1-1.10, 2.2-2.5, 3.1-3.4
IP1	1.7, 3.1-3.2, 8.1-8.7
IP2	1.7, 3.1-3.2, 8.1-8.7
IP3	1.4-1.6, 7.5
IP4	3.3, 5.13-5.20
IP5	1.7, 3.1-3.4, 7.7-7.9, 8.4-8.13
IP6	5.1-5.12, 6. 7.1-7.10, 9.1-9.13
IP7	8.1-8.13
IP8	4.1-4.2, 8.1-8.11
IP9	all items
IP10	all items
IM1	1.7-1.9, 2.4-2.5, 3.2-3.4, 4.5, 5.13-5.20, 7.6
IM2	1.7-1.9, 2.4, 5.1-5.12, 6, 7.1, 7.8, 8.8-8.13
IM3	1.6, 8.1-8.3, 8.5

All items were scored using the scale 0 - 4 with the exception of the items of the series 5 and 9 and items 8.8-8.13 which were scored dichotomously 0 - 1. The variable IP9 (the amount of drawings) was judges using the scale 1 - 5 and IP10 (time spent on tasks) was measured in minutes (exluding the time spent on reading the tasks). Scores 0 - 4 were given according to the following principle:

fluent performance without any hesitation or need of help 4p
correct performance after hesitation or some helping questions. 2p
no performance even after help 0p

The scores 1 and 3 were used only in some intermediate performances. Because it was difficult to fix the starting point of different subprocesses (in case of some students, in particular) only the total response time was used in the study with one exception. This exceptional variable was perceiving rate of geometric information in embedding context (IO2) with clearly restricted items, each consisting only of perceiving process. This variable IO2 can be considered as an interpretation of the concept of field-dependency in mathematics.

7. Results

7.1 "Primary" Abilities and Attitudes

The reliabilities of Werdelin´s ability tests have shown to be quite high, from .80 to .90 (Werdelin 1958, 97-100; Malinen 1969, 21). In this study the reliabilities were not estimated because the communalities of the tests in factor analysis showed to be of almost the same magnitude as in the former researches. The validity can be estimated by means of the control variable (school achievements = teachers´ marks). The "primary" abilities accounted for 52 % in the whole sample (N= 67) and 83 % in the process sample (N= 21) of the variance of school achievements in mathematics.

The factor structure of the mathematical abilities differed from that of Werdelin´s research with the inductive reasoning I and visualization Vz forming factors of their own. Summing up the variables which had the highest loadings on each factor and forming the second-order factors the following grouping was achieved:

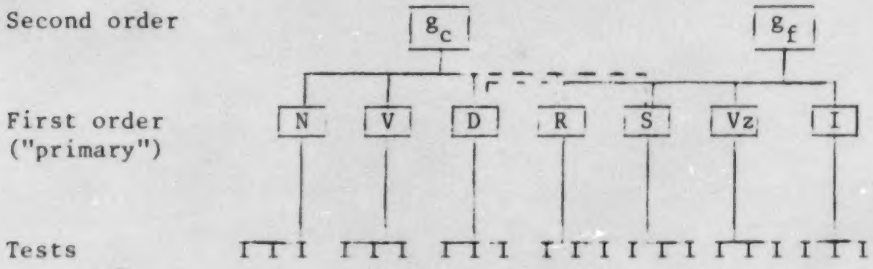
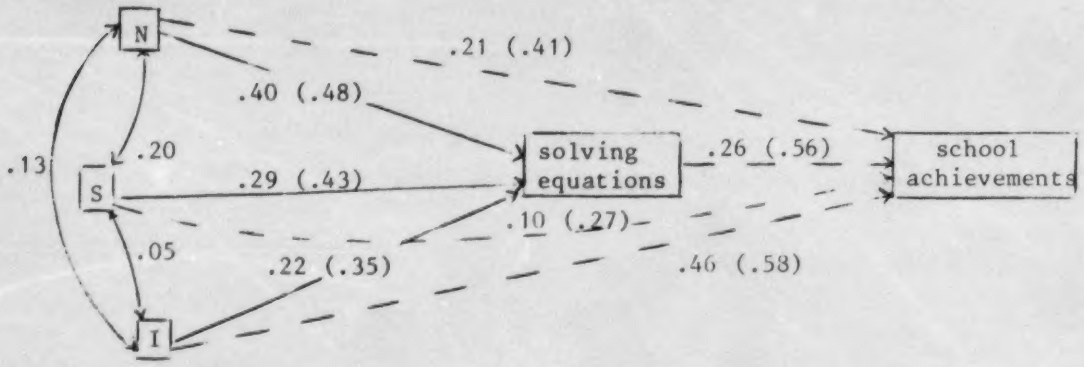


Figure 7.1 The structure of the mathematical abilities

The group factors g_c (crystallized intelligence) and g_f (fluid intelligence) are named according to Cattell´s theory (Cattell 1963, 1971). It should be noticed that D, R, and S had loadings of the same size class on g_c and g_f. The structure as a whole confirms very well previous findings (see e.g. the review of Treumann 1974).

The best explanatory variables in respect to school achievements were inductive reasoning I (r= .58), general reasoning R (.52), and numerical ability N (.41), and the worst ones were verbal ability V (.20) and spatial ability S (.27).

It was interesting to find out that solving equations of first degree was not related to purely numerical ability in the grade of the study as it was of major part in Werdelin´s research where older students were subjects. The path analysis showed that the best explanatory model of solving equation was the following



Numerical and spatial abilities and inductive reasoning accounted for 40 % of the variance of solving equation and all other "primary" abilities brought only 8 % more. We can conclude that before equation-solving has been automatized, it demands reasoning ability (what to do), spatial ability (moving terms from one side to the other and/or dividing by coefficient of the unknown), and numerical ability (performing calculations). (Because equation-solving test weighted on many factors, it was omitted from further analysis. Together with the ability tests they accounted for 54.4 % of the variance of school achievements in mathematics.)

The retest reliability of the attitude test estimated by Malinen was as high as .93. Because the communalities in this study were of the same size as in Malinen´s study, the reliability was not estimated here. The factor analysis produced five factors of attitudes:

A1 enterprisingness
A2 difficulty (of math)
A3 criticism of teaching
A4 acceptance of mathematics
A5 satisfaction with one´s achievements

The correlations between "primary" abilities and attitudes were low in the whole sample (much higher in the group of good achievers, see 7.4). Only the difficulty of mathematics (A2) was connected with reasoning abilities (R, I, and D) (correlations were .53, .43, and .38), and the satisfaction with one´s achievements (A5) was connected with inductive reasoning (.39).

The attitude variables accounted for less of the variance of school achievements than the "primary" abilities, in the whole sample (N=67) only 40 % and in the process sample (N=21) 52 %.

Comparing girls with boys there were some noticeable differences: girls' verbal and numerical abilities were higher than those of boys, whereas boys were better than girl in general and inductive reasoning. These results are in good agreement with former findings. Girls experienced mathematics more difficult than boys and their acceptance of mathematics was lower than that of boys.

The low achievers, compared with the high achievers, differred, in particular, in reasoning abilities (I, D, and R) and in numerical ability (N) whereas verbal ability was quite similar in both groups. Of course, mathematics was felt very difficult (A2) by the low achievers. (More detailed analysis in Leino 1977).

7.2 Information-Processing Abilities

The validation of the information-processing testing refers, in particular, to the totality of the costructs of the test. There are two different means to control the costruct validity in this study: one is the teachers' assessments about the mathematical abilities of the students and, the other, the psychometric test results. Every processing ability need not correlate highly with the teachers' assessments because the student can compensate some weak ability by other abilities.

The processing variables used in the study accounted for 76.5 % of the variance of teachers' marks. This can be considered as good evidence of the validation of the test. As the other criterion of the validation, 46.4-76.7 % of the variances of the "primary" abilities (N, R,...,V) were accounted for by the processing variables. The average amount accounted for by the processing variables was 62.8 %; the highest amounts were accounted for in the abilities of general and inductive reasoning, and the lowest amounts in verbal ability and visualization. The canonical correlation squared R^2_{c1} between the two linear combinations of the variable groups was .97.

To estimate the reliability of the process testing only split-half method was used. This method produced a very high reliability index of .96. Thus both reliability and validity indexes can be considered to be further evidence of the relevance of the results of the study. (The means and standard deviations of the process variables are given in Appendix 3.)

Some process variables were highly correlated with each other. Variables like these were information-obtaining (IO1, IO3, IO4) and retaining (IM1, IM2, IM3). Because the number of subjects was quite small the former were combined to form the obtaining variable IO´ and the latter the retaining one IM´ . Also the variables IP5 (drawing inferences), IP6 (generalizing objects, relations, operations), and IP7 (changing the direction of reasoning) were summed up as reasoning variable IP´ in further analysis because of high correlations. (The correlation matrices of the original and new variables are given in Appendices 3 and 4.)

Only two processing variables did not have significant correlations with other variables: IP1 using trial-and-error method and IP9 using drawing. These variables correlated even negatively with school marks in mathematics. This was consistent with Kilpatrick´s findings. The highest correlation .91 was between information obtaining IO´ and reasoning IP´ .These variables correlated quite highly with all other information-processing variables as well (with the exception of IP1 and IP9), especially with retaining information IM´ (.86 and .88). Thus retaining and recalling mathematical information seems to be in a close relationship with information obtaining and reasoning, and also with operating with symbols IP4. From the high negative correlations between processing time IP10 and other variables we can draw the conclusion that the processing time (including thinking and performing) is in a close relationship with other processing abilities.

By considering the correlations between school marks (SM) and processing variables we can conclude that reasoning IP´, remembering IM´, obtaining IO´ , and curtailing reasoning IP8 are the closest relationship with school achievements in mathematics. Also the field-dependency variable IO2 explained 30 % of the variance of school marks. The stepwise regression model of school marks is presented in table 7.2.1.

Variable	Stepwise β coefficients										
IP´	.77	.91	1.25	1.15	1.22	1.14	1.21	1.21	1.30	1.26	1.26
IP2		-.29	-.28	-.27	-.26	-.18	-.17	-.16	-.15	-.16	-.17
IO´			-.38	-.43	-.36	-.28	-.24	-.18	-.13	-.08	-.08
IP8				.19	.30	.44	.43	.38	.36	.37	.36
IP10					.26	.40	.40	.51	.55	.55	.57
IP9						-.21	-.23	-.25	-.25	-.24	-.23
IP3							-.12	-.19	-.21	-.19	-.20
IO2								.18	.20	.21	.23
IM´									-.12	-.15	-.17
IP1										-.06	-.04
IP4											.05
R^2(%)	58.8	65.2	67.8	69.6	71.4	74.4	75.0	76.0	76.2	76.5	76.5
F	27.1	16.9	11.9	9.2	7.5	6.8	5.6	4.8	3.9	3.3	2.7
df	1;19	2;18	3;17	4;16	5;15	6;14	7;13	8;12	9;11	10;10	11;9

Table 7.2.1 School marks as dependent variable and processing variables as independent variables in stepwise regression with β coefficients, R^2´s and F ratios.

Because of high intercorrelations between process variables individual coefficients are not interpreted.

7.3 "Primary" Abilities and Information-Processing Abilities

To discover relationships between "primary" abilities measured by psychometric tests and information-processing abilities measured by processing variables, the former are first considered individually interms of the latter.

Processes underlying numerical ability N consists of operations in short-term memory STM, and retrieving processes which bring relevant associations from long-term memory LTM. Processes like these were included only in a limited amount in the process tasks of the study. It was found out that those students who tried to solve tasks by using only their numerical skills were eager to guess a possible solution and try that. Thus it was not suprising that the regression model of numerical ability N consisted of trial-and-error variable IP1 as the best explanatory variable. Other independent variables that appeared in the model were reasoning variable IP´ and forming equation IP2:

N = .36 IP1 + .52 IP´ - .44 IP2 .

These three process variables accounted for 36 % of the variance of numerical ability, and all other process variables increased the total account by 32 %. Those students who attemted to form equation were not eager to guess and the minus sign in regression is evidence of that (also $r_{N,IP2}$ was negative).

General reasoning R is involved in abstract logical relations and quantitative algorithms of LTM according to Carroll´s analysis (1976, 44-45). In mathematics it usually means that the student has to understand the macro-structure of the problem and to make inferences on the same level. In this study R correlated highly with reasoning process IP´(.73), processing time IP10 (-.66), curtailing reasoning process IP8 (. 64), and obtaining information IO´ (.62) - in the last-mentioned sum variable especially with grasping formal structure of the task I04 (.60). All process variables accounted for 76.7 % of the variance of general reasoning of which the three best explanatory variables already accounted for 72.7 % :

$$R = .49\ IP' - .41\ IP9 + .42\ IP8 \ .$$

The minus sign of the coefficient of drawing figures IP9 is evidence that good reasoners neither needed nor used figures (also $r_{R,IP9}$ was negative); they were also able to curtail reasoning processes IP8 .

Deductive reasoning D consists of tests of syllogistic reasoning and hence is called also Rs = syllogistic reasoning. Carroll regarded this ability to be connected with lexicosemantic and abstract-logical memory store wher from meanings and algorithms are retrieved. It also involves strategies.In process terms of the study D was related to processing time IP10 (-.56), curtailing reasoning IP8 (.54), and field-dependence I02 (.49). Three best explanatory variables accounted for 55.8 % of the variance of D whereas the amount explained by all process variables was 69.7 % :

$$D = .11\ IP10 - .51\ IP9 + .59\ IP8 \ .$$

Inductive reasoning I is usually measured by rule forming tests (e.g. continuing number series). According to Carroll, the most important processes underlying induction are searching hypotheses from LTM and constructing new hypotheses. Thus, inductive reasoning involves some complex strategies as well. All this conforms to our study, because the closest prosesses related to induction were drawing inferences IP5, forming generalizations IP6, changing

direction of reasoning IP7 and recalling problem types IM3 (correlations over .60). The process variables accounted for 75.1 % of the variance of induction of which the three best explanatory variables (IP´, IO´, IP2) accounted for 66.7 %. It was interesting to notice that induction had a moderate correlation .56 with field-dependency, i.e. rule discovering overlapped figure discovering in complex context.

Spatial ability S is measured by tests involving mental rotation or moving of rigid configurations. Mental rotation of plain figures is a question of just STM but that of space figures involves also organizing, sequencing, and executing strategies and hence LTM. Three best explanatory process variables in the regression analysis of S, were getting expression of solution IP3, forming equation IP2, and using trial-and-error method IP1. They accounted for 38.6 % of the variance of S, whereas all process variables did 48.4 % .

Visualization Vz is usually measured by tests where some parts of a certain configuration are moved. Thus visualization contains additional (serial) mental operations compared with S. It demands strategies similar to rotation of space figures. In our study Vz had the highest correlations with processing time IP10 (-.62), field-dependency IO2 (.59), reasoning processes IP´´ (.56), and information obtaining IO´ (.55). The process variables accounted for 55.4 % of the variance of Vz. It is interesting to notice that the correlation between field-dependency IO2 and Vz is much higher that between IO2 and S. This conforms to the conception that visualization and field-dependency involve more complex and farther-reaching mental processes than spatial ability.

Verbal ability V tests measure meanings of words and vocabulary. Thus V depends upon the content of lexicosemantic LTM store (Carroll 1976, 44-45). The correlations between V and processing variables of the study were very low. Even the highest correlation was only .23 between V and reasoning variable IP´ . The process variables together accounted for 46.4 % of the variance of V. It is noteworthy that V did not correlate with processing time which contradicts to Hunt´s research results (Hunt et al. 1973).

The group of "primary" abilities can be regarded as one descriptive system of mathematical capabilities and information-processing variables as the other. By canonical analysis one group can be compared with the other. Table 7.3.1 presents the first canonical correlation and the structure coefficients of

variables. R_c^2 = 97.2 % was not significant but, leaving the worst explanatory variables IP1 and IP9 off, it would have been significant.

Structural coefficients				Canonical correlation
N	.05	IO´	.68	
R	.73	IO2	.72	R_c = .98
D	.49	IP1	-.36	
I	.80	IP2	.40	R_c^2 = 97.2 %
S	.43	IP3	.70	
Vz	.60	IP4	.77	Wilks Λ = .00036
V	.17	IP´	.87	
		IP8	.67	χ^2 = 91.3
		IP9	-.06	
		IP10	-.73	df = 77
		IM´	.74	

Table 7.3.1 Canonical analysis of "primary" abilities and process variables

From the analysis, an inference can be drawn: the common variance of the groups originates from the variables (I, R, Vz) and (IP´, IP4, IM´, IP10, IO2) where mathematical reasoning and information abstraction come to view. The process variables accounted for only 27.6 % (redundance index) of the total variance of "primary" abilities through the canonical variables because only 0.3 % and 2.9 % of the variances of N and V were accounted for, respectively. In the inverse direction the "primary" abilities accounted for 41.1 % of the total variance of the process variables. We can conclude that the process tasks of the study are insufficient to measure every process which demands numerical and verbal abilities.

What "primary" abilities are best related with individual information-processing variables can be considered on the bases of correlations.

Mathematical information obtaining IO´ provides general reasoning R (r= .62), visualization Vz (.55) particularly in geometric situations, and inductive reasoning I (.53). Perceiving geometric information in embedded context IO2 was related to visualization Vz (.59), induction I (.56), and also other reasoning abilities R and D (both .49).

Using trial-and-error was in small amount related to numerical ability N (.34) and the lack of e.g. inductive reasoning I (-.33). Forming and using equation and similar processes IP2 was related to spatial and visualizing abilities and general reasoning. Getting the expression for solution IP3 was related to reasoning abilities (R .58, I .55) and visualizing abilities (Vz .53,

S .49). Operating with symbols and numerical IP4 implied again reasoning abilities (R .54, I .53) and visualization (Vz .53). Drawing inferences, making generalizations and changing reasoning direction IP´ depended on general reasoning R (.73), induction I (.72), and visual abilities Vz and S (.56, .45). Curtailing reasoning processes IP8 was related to reasoning abilities (R .64, D .54, I .52) and visualization Vz (.53). Drawing figures IP9 was not positively related to any ability but negatively in small amount to reasoning abilities (D, R). Those students who could not make any progress started to draw some figures. Processing time IP10 was negatively correlated with most of "primary" abilities, in particular with R (-.66), Vz (-.62), D (-.62), and I (-.52). The more able the students were, in these abilities, the faster they made progress.

Retaining and retrieving mathematical information IM´ was related to induction I (.61) (rule forming memory), general reasoning R (.56) (problem type memory), and visualization Vz (.46) (visual memory).

Only some correlations between process variables and attitudes were significant: difficulty of mathematics A2 correlated significantly with curtailing reasoning process IP8 (.64), getting geometric information in diffuse situations IO2 (.57), and reasoning processes IP´ (.56). From this information we can infer that difficulty of mathematics originates from mathematical reasoning and obtaining information in complex situations. Because other correlations were quite low further considerations are omitted here.

7.4 Good, Average, and Poor Achievers

For comparing characteristics of good, average, and poor achievers the subjects were grouped using their school marks in mathematics. The scale of school marks, used in Finland, is 10 - 4. In the study, the marks 10 - 9 were regarded as good, 8 - 7 average, and 6 - 4 poor. The numbers of the subjects in the three groups formed were 6, 8, and 7 respectively. The means of the variables in the groups are given in table 7.4.1. There are also indexes referring to the significance of the differences when the average group is used as a basis for comparison.

Variable	Means good	average	poor	
N	587*	488	495	
R	607	515	453*	
D	617***	454	511	
I	609	529	404*	"primary"
S	555	506	483	abilities
Vz	546	516	469	
V	529	485	446	
A1	328	308	324	
A2	404*	308	254*	
A3	353**	267	290	attitudes
A4	244	258	290	
A5	311	304	257	
IO´	132	128	95*	
IO2	100*	70	61	
IP1	2.0	5.5	5.4	
IP2	2.0	6.3	1.1*	information-
IP3	12.7	10.4	6.9	processing
IP4	8.3	8.4	6.0*	variables
IP´	111	98	67**	
IP8	10.8	9.25	6.0*	
IP9	2.3	3.5	2.6*	
IP10	56.3	66.9	76.1	
IM´	91.2	84.4	69.3*	

Table 7.4.1 The means of good, average, and poor achievers in mathematics

Good achievers. From the table we can notice that a good achiever, compared with an average one, is superior in all "primary" abilities. Typically a good achiever is an excellent reasoner and good at calculations. Mathematics is not difficult for him and he is also satisfied with mathematics teaching. He accepts mathematics as an important school subject and is neutral as regards his achievements in math.

A good achiever is typically capable of perceiving mathematical information quickly, even in a complex context, and remembering terminology, concepts, generalizations, and problem types. He is usually, but not necessary, fast in his thinking and performing. He is good in dealing with symbols but not better than average students. He likes to solve problems by reasoning - and this was possible in almost all tasks of the study. Thus he is not eager to use drawings or equations if this is not necessary. He seldom uses trial-and-error method, but when he does it is because it is the shortest way to get an "obvious" solution and then he always attempts to give a reason for doing so. He is best at curtailing reasoning processes.

A good achiever is not a uniform type; there is a great diversity. The biggest differences exist in the domain of spatial ability and visualization or, in process term, in perceiving information in complex situations and using figures as aids. Of the six good achievers there were three students who can be called geometric type and tree other students who can be called analytic type. The two extreme representatives of these types had the following scores in different variables:

	N	R	D	I	S	Vz	V	IO´	IO2	IP1	IP2	IP3	IP4	IP´	IP8	IP9	IP10	IM´
geom.	600	643	664	708	559	579	454	145	154	4	0	16	7	122	20	5	92	33
anal.	658	526	632	609	446	504	448	130	62	8	0	13	8	96	8	2	87	59

The geometric student had good spatial ability and visualization, he perceived geometric information with ease, used visual aids, and liked to solve problems where he could draw figures. The other geometric students had respectively good visual abilities and they perceived geometric information easily, but they did not use visualizing aids or especially like to solve problems where figures could be drawn. The analytic students had visual abilities below the average, they did not like problems related to geometric context, but they had good reasoning and numerical abilities and they could analyse problems into pieces and sequences. The extreme students had verbal ability below the average whereas the non-extreme students were quite high in verbal ability. To conclude, we agree with Krutetskii (1976) that there exist two polar types of mathematically capable students, named geometric and analytic, and between these, one or two harmonic types.

It was noticeable that in the group of good achievers the structure of attitudes was clearly differentiated and correlations between attitudes and many abilities were very high or of moderate size. For instance, acceptance of mathematics A4 correlated with deductive reasoning D (.95), numerical ability N (.90), and general reasoning R (.71); criticism of teaching A3 correlated with spatial ability S (.86), general reasoning R (.83), and deductive reasoning D (.58), and difficulty of mathematics A2 with deductive reasoning D (.84), general reasoning R (.71), and numerical ability N (.57). Attitudes of good achievers correlated highly also with process variables as well.

Poor achievers. A poor achiever was not poor in all "primary" abilities. His numerical ability and deductive reasoning can be at least as good as those of an average student, but his general and inductive reasoning were inferior

and visual abilities weak. Mathematics was difficult for him but he did not criticize teaching. He was not at all satisfied with his achievements.

The information processes of a poor achiever clearly revealed the weak points of his performing. He is slow, uncertain, and his reasoning is not effective enough to discover the macro-structure of the problem. Because he is usually field-dependent he cannot analyze the problem e.g. form an equation or draw a figure. His knowledge of mathematics is inferior compared to that of an average achiever.

The diversity of abilities, attitudes, and processes of poor achievers was equal to that of good achievers. Some students of the poor group were above the average in spatial ability and visualization, and some other students in verbal and numerical abilities. Those above average in verbal ability, had numerical ability above average, as well, and vice versa.

8. Discussion

The study consisted of a theoretical scrutiny of traditional psychometric tests, cognitive processes, and their interrelationships, and an empirical application of the theoretical considerations on the level of junior secondary school. The context of the empirical part was offered by school mathematics.

Psychometric testing has been used to reveal mental processes underlying test performance. With increased experiences in the use of the tests the disadvantages, however, became more apparent. Especially intelligence tests were regarded as too rough to be of any use for education. Though intelligence is education´s most important product (Snow 1980, 185) it has been meaningless for practical educators. The differentiated concept of intelligence, such as basic dimensions or factors of mental ability, was closer to school education but, still, there was a lack of the dynamics of mental processes that could bind the factors to different school subjects, teaching of the subjects, and finally individual students´ constitutions and life experiences. There is little theoretical interest in any factor, unless one can offer an account of why the various performances loaded on the factor are correlated or otherwise linked (Carroll 1978, 85).

If the performances required in mental ability test are seen as based on learned and developed generalized abilities, it would be useful to assess the extent to which a student has acquired these abilities and what can be done to develop these abilities further. The developing process requires a lot of basic research of the possibilities to affect the growth of an individual´s abilities.

The test on a certain "primary" ability, e.g., has been believed to be homogeneous without further investigation. Usually the total test score of an individual is a sum of his correct responses and the individual differences associated with some variations of testing conditions or designs are treated as measurement unreliability. To use a number of component scores instead of a single score for each subject has been beneficent (Goldberg 1979). Cognitive theory represents new means to interpret basic mental abilities in terms of learning, memory, problem solving, strategies etc. So far, the research results of cognitive processes have been rather primitive as compared with the conceptual level of school teaching.

The empirical approach of the study aimed at discovering relationships between "primary" abilities in terms of psychometric factor constructs, and thinking and performing processes such as they occur in tasks of school mathematics. The processing variables used were selected according to the previous studies and theories about problem-solving processes in mathematics. An attempt was made to construct a prescriptive system for mathematical problem-solving. The processing abilities including approaching strategies (heuristics) and mathematical operations and generalizations were assessed in tasks where they were likely to appear as clearly as possible.

It was found out that the prescriptive system of problem-solving accounted for 76.5 % of the variance of students´ achievements in mathematics (school marks). Thus the construct validity of the system can be regarded to be quite good. Also the reliability seemed to be sufficiently high.

Of the processing variables reasoning, obtaining and remembering information, and curtailing reasoning processes had the highest correlations with achievements (correlation coefficients varied from .77 to .62). Two processing variables, namely trial-and-error and using figures, were not significantly related to achievements. Information retaining and recalling seemed to be in a close

relationship to information obtaining (perceiving) and reasoning (drawing inferences, generalizing objects, relations and operations, changing direction of reasoning) and these three variables to all other processing variables (with the exception of trial-and-error and drawing figures). Information retaining was also connected with operating with numerical and other symbols. Also processing rate was closely related to other variables but only moderately to global achievements in mathematics.

The processing variables accounted for 76.7-46.4 % of the variances of the "primary" abilities, best of the variances of general reasoning (77 %), induction (75 %), and deduction (70 %), worst of verbal (46 %) and spatial (48 %) abilities. In the continuation investigations spatial tasks should be increased, and verbal processes, such as reading, should be controlled.

Perceiving mathematical information was closely related to visualization, induction, and general reasoning. Trial-and-error method was used by the students who lacked inductive reasoning and tried to solve problems just by numerical ability. Forming and using equations were related to spatial ability, visualization, and general reasoning. It was found out that in the stage where solving equations is not a routine process, the best explanatory variables were inductive reasoning (what to do), spatial ability (moving terms from one side to the other and dividing by the coefficient of the unknown), and numerical ability (performing calculations).

Operating with symbols, drawing inferences, making generalizations, changing reasoning direction, and curtailing reasoning processes were related especially to general reasoning, induction, and visualization. Processing rate was related to all reasoning abilities and visualization. Retaining and recalling information was related to induction (rule forming memory), general reasoning (problem type memory), and visualization (visual memory).

Good achievers in mathematics were distinguished from average achievers: they had better reasoning abilities, they perceived information better even in complex situations, and they considered mathematics easy and did not criticize mathematics teaching. A good achiever was not a uniform type but many types of mathematical cast of mind were noticed. The main dimension of the types was geometric-analytic which confirmed Krutetskii´s findings (1976).

A poor achiever was not poor in all abilities. Some poor achievers had rather good numerical ability and deduction but general and inductive reasoning were

weak, as well as visualization. Those poor achievers who had weak verbal ability seemed to have weak numerical ability, as well. Mathematics was regarded difficult but teaching was not criticized. The information processes revealed weak points clearly: a poor achiever was slow, uncertain, and his reasoning was ineffective to discover macro-structures of tasks. Usually he was field-dependent and could not analyze problems in detail.

Although the sample in the study was rather representative, it was small. This was the main reason why e.g. teacher characterictics were omitted. It should also be noticed that speaking-aloud technique cannot reveal all thinking processes and that the quality of the technique depends on subjects. Not all students are verbal, and usually only the good achievers can verbalize their thinking processes in sufficient detail. An additional problem is that it is not certain if the student can verbalize the actual route by which he got a solution. He can tell only those parts which have come into consciousness and not until the moment when some results of cognitive processes appear (see Nisbett and Wilson 1977). The metacognitions that refer to the subject´s awareness of his thinking and executing processes and strategies may be the most important. They may also be the ones that the student finds the most difficult to verbalize about. Thus speaking-aloud technique might be complemented with tasks by means of which the investigator can assess metacognitions unconsciously revealed by the subject.

After the processes, strategies, and metacognitions used by students have been discovered, the following process will be investigating how an individual student can be helped to monitor the cognitive operations of all levels. (For the present situation in this field of study see e.g. Meichelbaum 1980.)

References

Anderson, J.R.,Bower, G.H. 1972. Recognition and retrieval processes in free recall. Psychological Review 79, 97-123.

Atkinson, R.C., Shiffrin, R.M. 1968. Human memory: A proposed system and its control process. In K.W. Spence, J.T. Spence (eds.): The Psychology of Learning and Motivation: Advances in Research and Theory. Vol. 2, 89-195. New York: Academic Press.

Ausubel, D.P. 1963. The Psychology of Meaningful Verbal Learning. New York: Grune & Stratton.

Bloom, B., Broder, L. 1950. Problem Solving Processes of College Students. Chicago: University of Chicago Press.

Brown, A.L. 1977. Knowing When, Where, and How to Remember: A Problem of Metacognition. (Tech. Rep.No.47). Illinois: University of Illinois, Center for Study of Reading.

Brown, A.L., DeLòache, J.S. 1978. Skills, plans, and self-regulation. In R.S. Siegler (ed.): Children´s Thinking: What Develops ?, 3-35. Hillsdale: Erlbaum.

Brown, W., Thomson, G. 1921. The Essentials of Mental Measurement. Cambridge: Cambridge University Press.

Bruner, J.S., Goognow, J.J., Austin, G.A. 1956. A Study of Thinking. New York: Wiley.

Burt, C. 1963. Intelligence distributed normally ? British J. of Statistical Psychology 16, 175-190.

Butterfield, E.C., Wambold, C., Belmont, J.M. 1973. On the theory and practice of improving short-term memory. American J. of Mental Deficiency 77, 654-669.

Carpenter, P.A., Just, M.A. 1978. Eye fixations during mental rotation. In J. Senders, R. Monty, D. Fisher (eds.): Eye Movements and Psychological Processes II. Hillsdale: Erlbaum.

Carroll, J.B. 1968. Reviews: Guilford, J.P. The Nature of Human Intelligence. American Educational Research J. 5, 249-256.

Carroll, J.B. 1976. Psychometric tests as cognitive tasks: A new "Strucuture of Intelligence". In L.B. Resnick (ed.): The Nature of Intelligence, 27-56. Hillsdale: Erlbaum.

Carroll, J.B. 1978. On the theory-practice interface in the measurement of intellectual abilities. In P. Suppes (ed.): Impact of Research on Education: Some Case Studies, 1-105. Washington: National Academi of Education.

Cattell, R.B. 1963. Theory of fluid and crystallized intelligence: A critical experiment. J. of Educational Psychology 54, 1-23.

Cattell, R.B. 1971. Abilities: Their Structure, Growth, and Action. Boston: Houghton Mifflin.

Charleswoth, W.R. 1976. Human intelligence as adaptation: An ethological approach. In L.B. Resnick (ed.): The Nature of Intelligence, 147-168. Hillsdale: Erlbaum.

Chi, M.T.H. 1976. Short-term memory limitations in children: Capacity or processing deficits ? Memory & Cognition 4, 559-572.

Chi, M.T.H. 1978. Knowledge, structures and memory development. In R.S. Siegler (ed.): Children´s Thinking: What Develops ?, 73-96. Hillsdale: Erlbaum.

Chiang, A., Atkinson, R.C. 1976. Individual differences and interrelationships among a select set of cognitive skills. Memory & Cognition 4, 661-672.

Cohen, G. 1977. The Psychology of Cognition. London: Academic Press.

Collins, A.M., Quillian, M.R. 1969. Retrieval time from semantic memory. J. of Verbal Learning and Verbal Behavior 8, 240-247.

Cooper, L.A., Shepard, R.N. 1973. Chronometric studies of the rotation of mental images. In W.G. Chase (ed.): Visual Information Processing. New York: Academic Press.

Cronbach, L.J. 1951. Coefficient alpha and the internal structure of tests. Psychometrica 16, 297-334.

Das, J.P., Kirby, J.R., Jarman, R.F. 1979. Simultaneous and Successive Cognitive Processes. New York: Academic Press.

Duncker, K. 1945. On problem solving. Psychological Monographs: General and Applied 58, No. 5. Washington.

Eysenck, H.J. 1939. Review of primary abilities. British J. of Educational Psychology 9, 270-275.

Flavell, J.H. 1977. Cognitive Development. Englewood Cliffs: Prentice Hall.

Flavell, J.H. Wellman, H.M. 1977. Metamemory. In R.V. Kail, J.W. (eds.): Perspectives on the Development of Memory and Cognition, 3-33. Hillsdale: Erlbaum.

Gagné, R.M. 1962. The acquisition of knowledge. Psychological Review 69, 355-365.

Gagné, R.M. 1977 (3rd ed.): The Condition of Learning. New York: Holt, Rinehart and Winston.

Gagné, R.M., Briggs, L.J. 1979 (2nd ed.): Principles of Instructional Design. New York: Holt Rinehart and Winston.

Gagné, R.M., White, R.T. 1978. Memory structures and learning outcomes. Review of Educational Research 48, 187-222.

Gardner, R.W. 1953. Cognitive styles in categorizing behavior. J. of Personality 22, 214-233.

Gardner, R.W., Long, R.I. 1962a. Control defense and centration effect: A study of scanning behavior. British J. of Psychology 53, 129-140.

Gardner, R.W., Long, R.I. 1962b. Cognitive control of attention and inhibition: A study of individual consistenies. British J. of Psychology 53, 381-388.

Goldberg, L.R. 1979. A general scheme for the analytic decomposition of objective test scores: Illustrative demonstrations using the rod-and-frame test and Müller-Lyer illusion. J. of Research in Personality 13, 245-265.

Goldin, G.A., McClintock, C.E. (eds.) 1980: Task Variables in Mathematical Problem Solving. Ohio State State University: ERIC.

Goodenough, D.R. 1976. The role of individual differences in field-dependence as a factor in learning and memory. Psychological Bulletin 83, 675-694.

Greeno, J.G. 1978. Natures of problem-solving abilities. In W.K. Estes (ed.): Handbook of Learning and Cognitive Processes. Vol. 5. Hillsdale: Erlbaum.

Guilford, J.P. 1976. The Nature of Human Intelligence. New York: McGraw-Hill.

Guilford, J.P. 1979. Cognitive Psychology with a Frame of Reference. San Diego: EdITs.

Haber, R.N., Standing, L.G. 1970. Direct estimates of apparent duration of a flash followed by visual noise. Cadanian J. of Psychology 24, 216-229.

Harvey, O.J., Hunt, D.E., Schroder, H.M. 1961. Conceptual Systems and Personalitity Organization. New York: Wiley.

Holzman, P.S., Klein, G.S. 1954. Cognitive system-principles of levelling and sharpening: Individual differences in visual time-error assimilation effects. J. of Psychology 37, 105-122.

Holzman, T.G., Glaser, R., Pellegrino, J.W. 1976. Process training derived from a computer simulation theory. Memory & Cognition 4, 349-356.

Horn, J.L. 1976. Organization of abilities and the development of intelligence. Psychological Review 75, 242-259.

Hunt, D.E. 1971: Matching Models in Education. Toronto: The Ontario Institute for Studies in Education.

Hunt, E., Frost, N. Lunneborg, C. 1973. Individual differences in cognition: A new approach to intelligence. In G.H. Bower (ed.): The Psychology of Learning and Motivation. Vol.7. New York: Academic Press.

Jensen, A.R. 1970. Hierarchical theories of mental ability. In B. Dockrell (ed.): On Intelligence. Toronto: Ontario Institute for Studies in Education.

Kagan, J., Moss, H.A., Sigel, I. 1963. The psychological significance of styles of conceptualization. In J.C. Wright, J. Kagan (eds.): Basic Cognitive Processes in Children. Monographs of the Society for Research in Child Development 28 (2), 73-112.

Kagan, J., Rosman, B.L., Day, D., Albert, J., Phillips, W. 1964. Information processing in the child: Significance of analytic and reflective attitudes. Psychological Monographs: General and Applied 78, No.1 (Whole No. 578).

Kantowski, G.M. 1975. Processes involved in mathematical problem solving. Dissertation Abstracts International 36, 2734A.

Keating, D.P., Bobbitt, B.L. 1977. Individual and developmental difference in cognitive processing components of mental ability. Child Development 49, 155-167.

Kelly, G.A. 1955: The Psychology of Personal Constructs. New York: Norton.

Kilpatrick, J. 1967. Analyzing the solution of word problems in mathematics: An exploratory study. Unpublished doctoral dissertation, Stanford University.

Kintsh, W. 1972. Notes on the structure of semantic memory. In E. Tulving, W. Donaldson (eds.): Organization of Memory, 247-308. New York: Academic Press.

Kintsh, W. 1975. Memory representation of text. In R.L. Solso (ed.): Information Processing and Cognition. The Loyola Symposium, 269-294. Hillsdale: Erlbaum.

Klein, G.S. 1951. Personal world through perception. In R.R. Blake, G.V. Ramsey (eds.): Perception: An Approach to Personality, 328-355. New York: Ronald.

Klein, G.S. 1958. Cognitive control and motivation. In G. Lindzey (ed.): Assessment of Human Motives. New York: Holt, Rinehart and Winston.

Kobasigawa, A. 1974. Utilization of retrieval cues by children in recall. Child Development 45, 127-134.

Kogan, N. 1976: Cognitive Styles in Infancy and Early Childhood. Hillsdale: Erlbaum.

Kotowsky, K., Simon, H.A. 1963. Human acquisition of concepts for sequential patterns. Reprinted in H.A. Simon (ed.) 1979: Models of Thought, 263-273. New Haven: Yale University Press.

Kotowsky, K., Simon, H.A. 1973. Empirical tests of a theory of human acquisition of concepts for sequential patterns. Reprinted in H.A. Simon (ed.)1979: Models of Thought, 274-291. New Haven: Yale University Press.

Kreutzer, M.A., Leonard, C., Flavell, J. 1975. An interview study of children's knowledge about memory. Monographs of the Society for Research in Child Development 40,(1, Serial No. 159).

Krutetskii, V.A. 1976: The Psychology of Mathematical Abilities in School Children. Chicago: University of Chicago Press.

Kuder, G.F., Richardson, M.W. 1937. The theory of the estimation of test reliability. Psychometrica 2, 151-160.

Kuhn, T.S. 1970 (2nd ed.): The Structure of Scientific Revolution. Chicago: The University of Chicago Press.

Kulm, G., Campbell, P.F., Frank, M., Tarsma, G. 1981. Analysis and Synthesis of Mathematical Problem Solving Processes. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, St. Louis, April 24, 1981.

Köhler, W. 1940: Dynamics in Psychology. New York: Livereight.

Lawson, M.J. 1980. Metamemory: Making decisions about strategies. In J.R. Kirby, J.B. Biggs (eds.): Cognition, Development, and Instruction, 145-159. New York: Academic Press.

Leino, A-L. 1980. Learning Process in Terms of Styles and Strategies: Theoretical Background and Pilot Study. Research Bulletin No. 54. Institute of Education, University of Helsinki.

Leino, J.A.Y. 1977. Matemaattisten Kykyjen ja Ajatteluprosessien Kehittäminen Kouluopetuksessa. 1. Matemaattiset Kyvyt ja Asenteet. Tutkimuksia N:o 60. Kasvatustieteen Laitos, Helsingin Yliopisto.

Leino, J.A.Y. 1978. Matemaattisten Kykyjen ja Ajatteluprosessien Kehittäminen Kouluopetuksessa. 2. Matemaattinen Ajattelu- ja Suoritusprosessi. Tutkimuksia N:o 66. Kasvatustieteen Laitos, Helsingin Yliopisto.

Lord, F.M. 1952 The relation of the reliability of multiple-choice test to the distribution of item difficulties. Psychometrica 17, 181-194.

Lord, F.M. 1970. Some test theory for tailored testing. In W.H. Holzman (ed.): Computer-Assisted Instruction, Testing and Guidance. New York: Harper & Row.

Lord, F.M., Novick, M.R. 1968: Statistical Theories of Mental Test Scores. Reading: Addison-Wesley.

Lucas, J.F. 1972. An explanatory study in the diagnostic teaching of elementary calculus. Dissertation Abstracts International 32, 6825A.

Luria, A.R. 1966a: Higher Cortical Function in Man. New York: Basic Books.

Luria, A.R. 1966b: Human Brain and Psychological Processes. New York: Harper & Row.

Luria, A.R. 1973: The Working Brain. London: Penguin.

Luria, A.R. 1976. The Neuropsychology of Memory. New York: Wiley.

Malinen, P. 1969: The Learning of Elementary Algebra. Research Bulletin No. 25. Institute of Education, University of Helsinki.

Mayer, R.E. 1977: Thinking and Problem Solving: An Introduction to Human Cognition and Learning. Glenview: Scott, Foresman & Co.

McClintock, C.E. 1980. Heuristic processes as task variables. In G.A. Goldin, C.E. McClintock (eds.): Task Variables in Mathematical Problem Solving. Ohio State University: ERIC.

McKinney, J.D. 1975. Problem-solving strategies in reflective and impulsive children. J. of Educationa' Psychology 63, 807-820.

Meichenbaum, D. 1980. A cognitive-behavioral perspective on intelligence. Intelligence 4, 271-283.

Messer, S.B. 1976. Reflection-impulsivity: A review. Psychological Bulletin 83, 1026-1053.

Messick, S. (ed.) 1976: Individuality in Learning: Implications of Cognitive Styles and Creativity for Human Development. San Francisco: Jossey-Bass.

Miller, G.A. 1956. The magical number seven, plus or minus two: Some limits on our capacity for processing information. Psychological Review 63, 81-97.

Moursund, J.P. 1976. Learning and the Learner. Monterey: Brooks.

Newell, A., Simon, H.A. 1972: Human Problem Solving. Englewood Cliffs: Prentice-Hall.

Paivio, A. 1969. Mental imagery in associative learning and memory. Psychological Review 76, 241-263.

Paivio, A. 1976. Imagery in recall and recognition. In J.A. Brown (ed.): Recall and Recognition, 103-129. London: Wiley.

Pask, G. 1978: The Generality of Problem Solving. New Methods of Assessment and Stronger Methods of Curriculum Design. Final report,Vol.1. Institute of Educational Technology. The Open University. Milton Keynes.

Pask, G., Scott, B.C.E. 1972. Learning strategies and individual competence. International J. of Man-Machine Studies 4, 217-253.

Pearson, K. 1896. Mathematical contributions to the theory of evolution. III Regression, heredity and panmixia. Philosophical Transactions A 187, 253-318.

Pellegrino, J.M., Glaser, R. 1979. Cognitive correlates and components in the analysis of individual differences. Intelligence 3, 187-214.

Pettigrew, T.F. 1958. The measurement and correlates of category width as a cognitive variable. J. of Personality 26, 532-544.

Piaget, J. 1970: Structuralism. New York: Basic Books.

Polya, G. 1957: How to Solve It. Garden City: Doubleday Anchor Books.

Polya, G. 1962, 1965: Mathematical Discovery. Vol. 1 & 2. On Understanding, Learning, and Teaching Problem Solving. New York: Wiley.

Rasch, G. 1961. On general laws and the meaning of measurement in psychology. Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics IV, 321-334. Berkeley: University of California Press.

Royce, J.R. 1973. The conceptual framework for a multi-factor theory of individuality. In J.R. Royce (ed.): Multivariate Analysis and Psychological Theory, 305-407. London: Academic Press.

Rubinstein, M.F. 1974: Patterns of Promlem Solving. Englewood Cliffs: Prentice-Hall.

Rumelhart, D:E:, Lindsay, P:H:, Norman, D.A. 1972. A process model for long-term memory. In E. Tulving, W. Donaldson (eds.):Organization of Memory, 198-246. New York: Academic Press.

Sakitt, B. 1976. Iconic memory. Psychological Review 83, 257-276.

Schlesinger, H.J. 1954. Cognitive attitudes in relation to susceptibility to interference. J. of Personality 22, 354-374.

Schriven, M. 1980. Prescriptive and descriptive approaches to problem solving. In D.T. Tuma, F. Reif (eds.): Problem Solving and Education: Issues in Teaching and Research, 127-139. Hillsdale: Erlbaum.

Shannon, C.E., Weaver, W. 1949: The Mathematical Theory of Communication. Urbana: University of Illinois Press.

Shepard, R.N., Metzler, J. 1971. Mental rotation of three-dimensional objects. Science 171, 701-703.

Shiffrin, R.M., Schneider, W. 1980. Controlled and automatic human information processing: II Perceptual learning, automatic attending, and a general theory. In J.G. Seamon (ed.): Human Memory. Contemporary Readings,24-41. New York: Oxford University Press.

Sigel, I., Jarman, P., Hanesian, H. 1967. Styles of categorization and their intellectual and personality correlates in young children. Human Development 10, 1-17.

Simon, H.A. 1979. Models of Thought. New Haven: Yale University Press.

Simon, H.A., Lea, G. 1974. Problem solving and rule induction. Reprinted in H.A. Simon (ed.) 1979: Models of Thought, 329-346. New Haven: Yale University Press.

Skemp, R.R. 1979: Intelligence, Learning, and Action: A Foundation for Theory and Practice in Education. Chishester: Wiley.

Skinner, B.F. 1954: Science and Human Behavior. New York: Macmillan.

Snow, R.E. 1980. Intelligence for the Year 2001. Intelligence 3, 185-199.

Spearman, C. 1904. General intelligence objectively determined and measured. American J. of Psychology 15, 202-293.

Sternberg, S 1969. Memory-scanning: Mental processes revealed by reaction-time experiments. American Scientist 57, 421-457.

Sternberg, S. 1977: Intelligence, Information Processing, and Analogical Reasoning: The Componential Analysis of Human Abilities. Hillsdale: Erlbaum.

Thorndike, E.L., Bregman, E.O., Cobb, M.V., Woodyard, E. 1926: The Measurement of Intelligence. New York: Teachers College, Columbia University.

Thurstone, L.L. 1938: Primary Mental Abilities. Chicago: University of Chicago Press.

Treumann, K. 1974: Leistungsdimensionen im Mathematikunterricht. Dissertation, Universität Hamburg.

Tulving, E. 1962. Subjective organization in free recall of "unrelated words". Psychological Review 69, 344-354.

Tulving, E. 1972. Episodic and semantic memory. In E.Tulving, W. Donaldson (eds.): Organization of Memory, 381-403. New York: Academic Press.

Tuma, D.T., Reif, F. (eds.) 1980: Problem Solving and Education: Issues in Teaching and Research. Hillsdale: Erlbaum.

Tyler, L.E. 1976. The intelligence we test - an evolving concept. In L.B. Resnick (ed.): The Nature of Intelligence, 13-26. Hillsdale: Erlbaum.

Vasgird, D.R. 1980. The cognitive style challenge to IQ. In H.F. Taylor: The IQ Game: A Methodological Inquiry into the Heredity-Environment Controversy, 218-225. New Brunswick: Rutgers University Press.

Vernon, P.E. 1950: The Structure of Human Abilities. London: Methuen.

Waters, W.M. 1980. Concept acquisition tasks. In G.A. Goldin, C.E. McClintock (eds.): Task Variables in Mathematical Problem Solving, 277-296. Ohio State University: ERIC.

Webb, N.L. 1975. An exploration of mathematical problem-solving processes. Unpublished doctoral dissertation, Stanford University.

Werdelin, I. 1958: The Mathematical Ability. Lund: Gleerup.

Werdelin, I. 1961: Geometrical Ability and the Space Factors. Lund: Gleerup.

Wickelgren, W.A. 1973. The long and the short of memory. Psychological Bulletin 80, 425-438.

Wickelgren, W.A. 1974: How to Solve Problems: Elements of a Theory of Problems and Problem Solving. San Francisco: Freeman.

Witkin, H.A., Lewis, H.B., Herzman, M., Machover, K., Meissner, P.B., Wapner, S. 1954: Personality Through Perception. New York: Harper & Row.

Witkin, H.A., Moore, C.A., Goodenough, D.R., Cox, P.W. 1977. Field-dependent and field-independent cognitive styles and their educational implications. Review of Educational Research 47, 1-64.

von Wright, J., Vauras, M., Reijonen, P. 1979: Oppimisen Strategiat Kouluiässä I. Tutkimuksen Viitekehys ja Esitutkimus. Turun Yliopisto. Psykologian Tutkimuksia N:o 33. Turun Yliopisto.

Attitude test

The questionary consists of a set of statements. Mark

1 if you strongly agree (with statement)
2 if you agree
3 if you annot say
4 if you disagree
5 if you strongly disagree

into the square beside each statement.

1) Mathematics is a difficult subject as compared with other subjects ☐
2) There could be more home exercises in mathematics ☐
3) The textbook in mathematics is not as interesting as textbooks in other subjects ☐
4) I do not like mathematics for I am afraid of failing ☐
5) I have really tried my best to succeed in mathematics ☐
6) Sometimes I am completely depressed by my poor achievements in mathematics ☐
7) My achievements in mathematics are poor compared with my abilities ☐
8) Mathematics is a dull subject as compared with other subjects ☐
9) I could try more in mathematics if needed ☐
10) Very often I do poorly in mathematics though I ought to do better ☐
11) I think the teaching in mathematics has progressed too fast ☐
12) I cannot understand why my achievements in mathematics are worse than in other subjects ☐
13) The studying of mathematics is inspiring ☐
14) As a rule I prefer to have other school work rather than mathematics ☐
15) There are too great demands in mathematics at school ☐
16) I have really done my best to get my home exercises ready ☐
17) There are many unnecessary topics in mathematics ☐
18) I am satisfied with my results in mathematics ☐
19) It is difficult to understand the principles in mathematics ☐
20) I am too lazy to study mathematics enough ☐
21) The studying of mathematics makes me irritable and restless ☐
22) I need more elementary exercises in mathematics ☐
23) Only the talented in maths can understand school mathematics ☐
24) There could be more difficult exercises in textbook ☐
25) I like better exercises where a figure can be drawn and progress made based upon it ☐
26) I think I have not tried to succeed in mathematics as well as other students ☐
27) If I tried my best I should succeed better than other students on the average ☐

Information-processing tasks

Series 1. Problems with unstated question

1.1 In two saving accounts there are 140 mk altogether. If 15 mk is taken from one account and put into the other, each will have the same amount. (What conclusion can you make in this task ?)

1.2 A long tub, length of which was 155m, was composed of 25 iron tubs. Some of the tubs had length of 5m each, the other had length of 8m each. (What conclusion can you make in this task ?)

1.3 The student had on his bank account 140 mk. He withdrew such a sum of money that the remaining sum was only fifth of that he withdrew. (What conclusion...)

1.4 Two persons had 28 mk altogether. The one had a mk. (What ...)

1.5 A student bought b pieces of 1 mk's stamps and four times as many 50 p's stamps. (Question and expression.)

1.6 The speed of the car is x km/h during the whole trip of 760km. (Question and expression.)

1.7 A right angle is divided into two adjacent angles so that the smaller angle is half of the larger. (Question and result.)

1.8 Through the endpoints of linesegment AB we draw two lines intersecting at point C so that the angles C and A are 50° each. (What ...)

1.9 The intersection point of the diagonals of a rectangle is at the distance of 20 cm from the shorter sides and of 6 cm from the longer sides. (What...)

1.10 A square is given. If you shorten one pair of the opposite sides 1.2 m each and the other pair correspondingly 1.5 m, you will get a rectangle which is 14.4 m^2 smaller than the square. (What...)

Series 2. Problems with incomplete information

2.1 A dog runs 30 m behind a fox. The length of one dog leap is 2 m while one fox leap is only 1 m. Find how far they run until the dog reaches the fox.

2.2 The teacher bought the class 80 booklets. Part of these were ordinary booklets and cost only 60 p each, the other were special at 120 p each. How much did the booklets cost altogether ?

2.3 How long is one side of the rectangle if the area is 36 m^2?

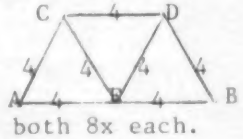
2.4 The distance between the centers of two circles is 10 cm. Find the shortest distance between the circles if the radius of one circle is 3 cm.

2.5 The sides of the triangle are proportional to numbers 3, 4, and 5. Find the length of the perimeter of the triangle.

Series 3. Problems with surplus information

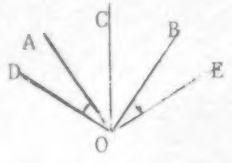
3.1 In a store, 24 sacks of potatos weigh 3 kg and 5 kg each, with more in the former ones than in the latter. The weight of all the 5 kg sacks was equal to the weight of all 3 kg sacks. How many sacks of each weight were in the store ? (Find the surplus information.)

3.2 In the figure there are three congruent equilateral triangles adjacent with sides of 4 cm each. Why are points A, E and B collinear ? (Find also the surplus information.)



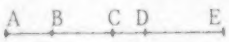
3.3 If the base of the isosceles triangle is 3x then the legs are both 8x each. The perimeter is 38 cm and the base is 10 cm shorter than the leg. Find the lebgth of the sides. (Find also the surplus information.)

3.4 OC is the bisector of the angle AOB. The angles DOA and BOE are congruent and equal to 20° each. Why is OC the bisector of angle DOE ? (Find also the surplus information.)



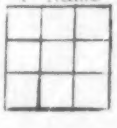
Series 4. Problems with interpenetrating elements

4.1 How many line segments can you see in the figure? Name them.

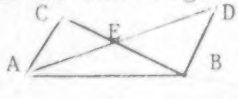


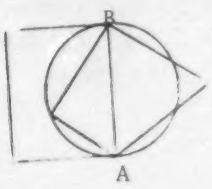
4.2 How many acute angles can you see in the picture ? Name them.

4.3 How many squares can you see in this figure ?



4.4 How many triangles can you see in this figure? Name them.





4.5 In this figure you can find a circle, a rectangle, triangles etc. What different names can you give line-segment AB according to the position it has in different figures.

Series 5 Rule applying

Inwhich one of the following cases can you apply the formula $(a+b)^2=a^2+2ab+b^2$:

5.1 $(a+x)^2$. 5.2 $(a+b).2$. 5.3 $a+b^2$. 5.4 a^2+b^2 . 5.5 $(m+n)^2$.

5.6 $(a+2b)^2$. 5.7 $(a+b^2)\cdot 2$. 5.8 $(a+b^2)^2$. 5.9 $(\frac{a}{2}+b)^2$. 5.10 $(6a^2+3b^4)^2$.

5.11 $(\frac{a}{2}+\frac{b}{3})^2$. 5.12 $(x^n+0.2y^m)^2$.

Apply the formula in the following cases:

5.13 $(a+x)^2=$ 5.14 $(m+n)^2=$

5.15 $(\frac{a}{2}+b)^2=$ 5.16 $(a+b^2)^2=$

5.17 $(\frac{a}{2}+\frac{b}{3})^2=$ 5.18 $(a+2b)^2=$

5.19 $(6a^2+3b^4)^2=$ 5.20 $(x^n+0.2y^m)^2=$

How can you easily find with help of the formula the value of

5.21 $51^2=$ 5.22 $102^2=$

Series 6. The level of abstraction. (The order of presentation is e,a,e,b,e,c,e,d,e)

6a. The length of one room is 6 m and both width and heigth are 3 m each. Find the volume of the apartment with 4 rooms of the same size.

6b. -"- 6 m -"- 3 m -"-
-"- n rooms -"-

6c. -"- 6 m, width 3 m and heigth b meters -"-
-"- n rooms -"-

6d. -"- 6 m and both width and heigth are b meters each. -"-
-"- n rooms -"-

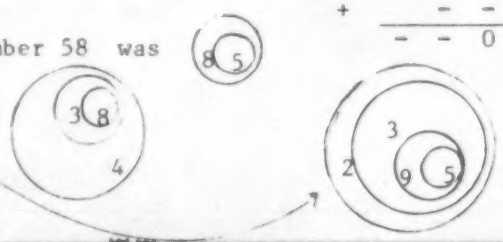
6e. -"- a meters, -"- b meters -"-
-"- n rooms -"-

Series 7. Generalization of objects

7.1 Is a always smaller than 3a ? If not, in what conditions isn't it ?
7.2 Is -d always negative ? -"- -"-
7.3 Can a be equal to 5a? If it can, in what conditions?
7.4 Can a+b be smaller than a-b ? -"- -"-
7.5 How can you give an expression for all even integers ?
7.6 The edge of a cube is 6 cm. Find the surface area of the cube.
7.7 There are 370 students in a school. Why do at least two students have the same birthday ?
7.8 If you draw a segment from the midpoint of one side to the opposite vertex, the area of the triangle is halved. Why ?
7.9 Complete the addition using nummers 0, 1, 2,...,9 instead of each fragment.

			4	-
+		-	-	2
	-	-	0	1

7.10 In the "circle country" the number 58 was written like this and 834 like this What number was written like



Series 8. Flexibility of thought

8.1 a) What is the sum if you add 14 to 18 ?

b) When you add 15 to a certain number you get 37. What is that number ?

8.2 a) The volume of a can is 80 l. How much is 2/5 of the volume of the can ?

b) 16 l is 2/5 of the volume of a can. Find the volume of the can.

8.3 a) The salary of a worker is a mk/h. How much does he get while working n hours?

b) The salary of a worker is a mk/h. How many hours is he to work to get c mk?

8.4 a) The production of a car factory is n cars in 3 days. How many cars does the factory produce in 5 days and how many in a days ?

b) The car factory produces n cars in 3 days. How many days does it take to produce 20 cars and how many to produce m cars ?

8.5 a) How much is 1/24 of the right angle ?

b) Which fraction of the right angle is the angle of 12° ?

8.6 When 5 is added to a certain number and the result is multiplied by 3, then the result is 21. Find the certain number.

8.7 When a certain number is divided by 2 and the quotient is added to 5, then the result is 9. Find the certain number.

The square of the difference of two numbers a and b is $a^2-2ab+b^2$.

8.8 Factor $x^2-2xy+y^2$. 8.9. Factor a^2-6a+9 . 8.10 Factor $a^2-2ab^2+b^4$.

The product of the sum and the difference of two numbers a and b is the difference of the squares of the numbers: $(a+b)(a-b) = a^2 - b^2$.

8.11 Factor x^2-y^2 . 8.12 x^2-9 . 8.13 $4a^2-9b^2$.

Series 9. Generalization of rules and operations

The operation $\spadesuit$ between a and b is defined by: $a \spadesuit b = a \cdot b + a - b$.

9.1 Find $4 \spadesuit 3$. 9.2 Find $x \spadesuit x$. 9.3 Is $a \spadesuit b$ equal to $b \spadesuit a$?

The sum 1+2+3+4+5+6+7+8 can be easily computed in this way: (1+8)+(2+7)+ +(3+6)+(4+5) = 4·9 = 36. Can you apply the method in the following problems ?

9.4 1+2+3+4+7+8+9+10 . 9.5 5+10+20+40+80+160 . 9.6 1+2+4+8+16+32

9.7 5+10+15+20+25+30 .

The method shown above can be transfer to multiplication. Find in that way the value of product

9.9 $4 \cdot 3 \cdot 2 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$.

9.10. Can you apply the method in the following case: $1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32$?

9.11 $5 \cdot 10 \cdot 20 \cdot 40 \cdot 80 \cdot 160$?

9.12 $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6$?

9.13 $1 \cdot 2 \cdot 3 \cdot 4 \cdot 6 \cdot 12$?

Table 1. The means and deviations of the information-processing variables

X_i	$\bar{x}$	s
IO 1	21.19	3.17
IO 2	75.52	33.35
IO 3	27.57	10.23
IO 4	69.62	15.06
IP 1	4.48	5.58
IP 2	3.33	4.99
IP 3	9.86	5.13
IP 4	7.57	2.50
IP 5	25.19	9.18
IP 6	43.62	9.64
IP 7	22.67	8.66
IP 8	8.62	4.03
IP 9	2.86	1.15
IP 10	66.95	21.09
IM 1	36.71	6.70
IM 2	27.24	7.09
IM 3	17.33	3.48
IO′	118.38	27.46
IP′	91.47	25.55
IM′	81.29	15.84

Table 2. The intercorrelations of the information-processing variables

IO 1															
IO 2	60														
IO 3	83	54													
IO 4	81	62	91												
IP 1	07	-03	02	-09											
IP 2	39	25	38	47	-19										
IP 3	60	58	66	80	-01	40									
IP 4	53	31	63	67	-48	64	58								
IP 5	77	64	83	88	-09	49	81	70							
IP 6	58	59	77	77	-42	44	62	79	77						
IP 7	77	57	86	84	-00	41	74	59	91	71					
IP 8	67	70	68	61	01	25	43	42	65	68	57				
IP 9	05	12	21	13	18	31	-05	-00	04	10	17	27			
IP 10	-75	-77	-77	-74	08	-36	-59	-61	-76	-76	-67	-78	-03		
IM 1	60	44	69	85	-18	55	73	73	77	66	75	34	12	-48	
IM 2	54	61	69	76	-33	42	51	65	75	86	68	62	18	-67	69
IM 3	79	56	82	88	-03	43	70	58	83	70	86	56	17	-61	83	81
	IO 1	IO2	IO3	IO4	IP1	IP2	IP3	IP4	IP5	IP6	IP7	IP8	IP9	IP 10	IM1	IM2

(Decimal points have been omitted)

Table 3. The correlation matrix of all variables

	IO′	IO2	IP1	IP2	IP3	IP4	IP′	IP8	IP9	IP10	IM′	N	R	D	I	S	Vz	V	A1	A2	A3	A4	A5	SM
IO′																								
IO2	61																							
IP1	-03	-03																						
IP2	44	25	-19																					
IP3	75	58	-01	40																				
IP4	66	31	-48	64	58																			
IP′	91	64	-03	48	78	75																		
IP8	67	70	01	25	43	42	68																	
IP9	16	12	18	31	-05	-00	11	27																
IP10	-78	-77	08	-36	-59	-61	-79	-78	-03															
IM′	86	58	-24	52	69	73	88	54	17	-64														
N	23	05	34	-26	17	-14	24	27	-21	-24	15													
R	62	49	-10	26	58	54	73	64	-25	-66	56	46												
D	36	49	-17	-13	32	25	40	54	-35	-56	37	55	61											
I	53	56	-33	10	55	53	72	52	-05	-52	61	18	53	34										
S	35	35	14	48	49	32	45	32	14	-30	38	13	50	24	-01									
Vz	55	59	-01	41	53	51	56	53	14	-62	46	-09	30	32	22	56								
V	07	-07	07	15	10	12	23	13	-16	-08	17	57	42	29	04	36	-15							
A1	-23	-19	20	-42	-05	-14	-18	-25	20	25	-17	03	-38	-20	15	-16	-06	-20						
A2	46	57	-15	03	34	23	56	63	-09	-38	49	32	74	48	61	31	19	15	-28					
A3	01	02	-34	-04	24	17	18	-03	-35	04	19	33	44	23	29	25	-15	32	12	45				
A4	-23	10	-13	05	-06	08	-24	-21	-12	-08	-30	-58	-36	-14	-11	-17	14	-33	22	-52	-34			
A5	08	08	08	09	09	12	09	11	-07	-01	04	01	28	-00	26	18	-02	07	03	37	17	01		
SM	62	54	-23	15	53	49	77	62	-08	-56	63	42	64	47	79	22	38	37	-04	69	37	-32	36	

$r = .42^{*}$

$r = .54^{**}$

IO′, IO 2,...,IM′ information-processing variables

N, R,..., V "primary" abilities

A1, A2,..., A5 attitudes

SM school mark (achievement)

Research Bulletin, Institute of Education, University of Helsinki

x No 1 The Effectiveness of Punishments and the School Climate by OLLI SIPINEN. May, 1957. 10 pp.

No. 2 The Balance of Instructional Situations by MATTI KOSKENNIEMI. Nov., 1957. (reprint 1972). 7 pp.

x No. 3 On Deceit Practiced in Finnish Grammar Schools at Written Tasks by OLAVI PÄIVÄNSALO. Nov., 1957. 5 pp.

No. 4 Comparison between Reading Comprehension and Reading Aloud by KAISA HÄLINEN. March, 1958. 9 pp.

x No. 5 Ten Test-Papers in the Swedish Language as Graded by Ten Teachers by VÄINÖ HEIKKINEN & ANITA HAVELA. Sept., 1958. 5 pp.

x No. 6 The Awarding of School Marks by TOIVO VÄHERVUO. Sept., 1958. 5 pp.

x No. 7 Some Comments on the Uniformity of the 'Experimental Period' in the History of Soviet Education by MATTI KOSKENNIEMI. Feb., 1959. 18 pp.

x No. 8 Motives for the Choice of the Teaching Profession Correlated with Success in the Entrance Examination and in Student Teaching by MATTI KOSKENNIEMI. Feb., 1960. 11 pp.

x No. 9 One's Own Teacher and the Choice of Teaching Profession by MATTI KOSKENNIEMI & EIVI LIHTONEN. Sept., 1960. 14 pp.

x No. 10 The Curriculum of School Mathematics and Mathematical Performance. An Experimental Study by PAAVO MALINEN. Sept., 1961. 50 pp.

No. 11 Prognostic Value of Some Psychological Test Results Correlated with Student Teaching Marks by MARJA-LEENA KÄYHTY. Dec., 1961. 10 pp.

x No. 12 Factors Constituting Classroom Climate by OLLI SIPINEN. March, 1963. 10 pp.

x No. 13 On the Changes in the Educational Attitudes of Young Teacher by VÄINÖ HEIKKINEN. March, 1963. 14 pp.

x No. 14 Teacher Training in Retrospection. A Preliminary Report by VÄINÖ HEIKKINEN. Feb., 1964. 21 pp.

No. 15 Placement of Young Elementary School Teachers by MATTI KOSKENNIEMI. May, 1965. (reprint 1972). 9 pp.

x No. 16 Some Problems in Combining the Individual Tutors' Marks into a Final Mark in Teaching Ability by JOHANNES ALIKOSKI. March, 1966. 25 pp.

No. 17 The Finnish Senior Secondary Research Project I. General Presentation of the Project and Its Original Subjects by ANNA-LIISA SYSIHARJU. Sept., 1967. 31 pp.

x No. 18 The Finnish Senior Secondary Research Project II. Departure from Junior Secondary and Transfer to Senior Secondary by ANNA-LIISA SYSIHARJU. Oct., 1967. 52 pp.

No. 19 Relationships between Absences from School and Certain Factors Related to School. A Study on Elementary School Grades Three to Six by PERTTI KANSANEN. Nov., 1967. 10 pp.

No. 20 On the Anxiety Associated with School Attendance and the Grammar School Entrance Examination by EEVA PATJAS. May, 1968. 16 pp.

No. 21 School Achievement and Personality. I: Design and Hypotheses, IV: Results and Discussion by ERKKI A. NISKANEN. Oct., 1968. 51 pp.

No. 22 School Achievement and Personality. II: Operations at the Variable Level by ERKKI A. NISKANEN. Oct., 1968. 124 pp.

No. 23 School Achievement and Personality. III: Operations at the Factor Level by ERKKI A. NISKANEN. Oct., 1968. 39 pp.

No. 24 The Effectiveness of Two Methods of Teaching English as a Foreign Language in Some Finnish Secondary Schools by DANIEL J. CASEY. Nov., 1968. 37 pp.

No. 25 The Learning of Elementary Algebra. An Empirical Investigation of the Results of Learning in a Simplified School Learning System by PAAVO MALINEN. Jan., 1969. 85 pp.

No. 26 Investigations into the Instructional Process. I. Some Methodological Problems by MATTI KOSKENNIEMI and ERKKI KOMULAINEN in collaboration with Anna-Kaarina Falck and Pentti Holopainen. Oct., 1969. (reprint 1972). 35 pp.

No. 27 Investigations into the Instructional Process. II. Objectivity of Coding in a Modified Flanders Interaction Analysis by ERKKI KOMULAINEN. Dec., 1970. 26 pp.

No. 28 Investigations into the Instructional Process. III. P-technique Treatment of Observational Data by ERKKI KOMULAINEN. Jan., 1971. 41 pp.

No. 29 Investigations into the Instructional Process. IV. Teaching as a Stochastic Process by ERKKI KOMULAINEN. Feb., 1971. 24 pp.

No. 30 Investigations into the Instructional Process. V. Experiences with the Bellack Classification System by KAI KARMA. Jan., 1972. 16 pp.

xNo. 31 Investigations into the Instructional Process. VI. Group Work as a Social Interaction Process. A Case Approach by PENTTI HOLOPAINEN. May, 1972. 34 pp.

No. 32 Investigations into the Instructional Process. VII. Non-verbal Behaviour in Group Work. Observational Technique and Typology of Pupils' Work by MARJA MARTIKAINEN. Sept., 1972. 18 pp.

xNo. 33 English School Achievements and Some Student Characteristics. I. On the Relationships of Personality and Intelligence Variables to English School Achievements by ANNA-LIISA LEINO. Oct., 1972. 50 pp.

No. 34 Investigations into the Instructional Process. VIII. On the Problems of Variable Construction from Flanders' Interaction Matrix with Special Emphasis on the Stochastic Nature of Classroom Communication by ERKKI KOMULAINEN. Feb., 1973. 78 pp.

No. 35 Zur Problematik des Unterrichts by CHR. PETROW CHRISTOW. April, 1973. 11 pp.

No. 36 Investigations into the Instructional Process. IX. Pupils' Goal-related Behavior during the Instructional Interaction by MATTI KOSKENNIEMI & PENTTI HOLOPAINEN. June, 1973. 17 pp.

xNo. 37 The Behaviour of Supervisors assessed by Student Teachers by SEPPO KONTIAINEN. Nov., 1973. 186 pp. Out of print.

No. 38 The Ability to Structure Acoustic Material as a Measure of Musical Aptitude. I. Background Theory and Pilot Studies by KAI KARMA. Nov., 1973. 19 pp.

xNo. 39 Investigations into the Instructional Process. X. Report of the DPA Helsinki, Phase One by MATTI KOSKENNIEMI & ERKKI KOMULAINEN. Feb., 1974. 35 pp.

xNo. 40 English School Achievements and Some Student Characteristics II. On the Relationships of Motivational and Auditory Variables to English School Achievements. Final Analyses by ANNA-LIISA LEINO. March, 1974. 48 pp.

No. 41 Foreign Language Curriculum. An Investigation into Objectives, Achievements and Student Characteristics by ANNA-LIISA LEINO. June, 1974. 27 pp.

No. 42 DPA Helsinki System for Describing Instructional Process. Manual. Prepared and edited on behalf of the Research Team DPA Helsinki by MATTI KOSKENNIEMI, KAI KARMA and MARJA MARTIKAINEN. Nov., 1974. 34 pp.

x No. 43 The Ability to Structure Acoustic Material as a Measure of Musical Aptitude. 2. Test Construction and Results by KAI KARMA. Feb., 1975. 21 pp.

No. 44 A Comparison of the Importance of Objectives in Foreign Language Instruction as Rated by In-Service Teachers, Student Teachers and Pupils by ANNA-LIISA LEINO. June, 1975. 35 pp.

No. 45 Implementation of Some Affective Objectives in Different Foreign Language Teaching Situations as Rated by Student Teachers by ANNA-LIISA LEINO. Nov., 1975. 45 pp.

No. 46 The Need Felt for In-Service Education by Foreign Language Teachers by ANNA-LIISA LEINO. March, 1976. 53 pp.

No. 47 The Ability to Structure Acoustic Material as a Measure of Musical Aptitude. 3. Theoretical Refinements by KAI KARMA. Aug., 1976. 32 pp.

No. 48 Structural Elements of the Instructional Process: Activity Forms and Content. DPA Helsinki Investigations I by MATTI KOSKENNIEMI. Dec., 1977. 36 pp.

No. 49 Research in Teaching. Papers presented at the symposion arranged by the Academy of Finland for evaluation of the DPA Helsinki Project, Helsinki, October 26th and 27th 1977. DPA Helsinki Investigations II edited by ERKKI KOMULAINEN and MATTI KOSKENNIEMI. June, 1978. 152 pp.

No. 50 An Introduction to the Content of Foreign Language Instruction by ANNA-LIISA LEINO . June, 1978. 80 pp.

x No. 51 The Ability to Structure Acoustic Material as a Measure of Musical Aptitude. 4. Experiences with Modifications of the Acoustic Structuring Test by KAI KARMA. Dec., 1978. 51 pp.

x No. 52 The Ability to Structure Acoustic Material as a Measure of Musical Aptitude. 5. Summary and Conclusions by KAI KARMA. March, 1980. 33 pp.

x No. 53 The Electronic Hand Calculator in Finnish Comprehensive School Mathematics Teaching by JOHANNES PAASONEN. Oct., 1980. 50 pp.

No. 54 Learning Process in Terms of Styles and Strategies: Theoretical Background and Pilot Study by ANNA-LIISA LEINO. Dec., 1980. 40 pp.

No. 55 Emotion and Cognition in Visual Art Education by INKERI SAVA. March, 1981. 46 pp.

No. 56 Classroom Analysis: Concepts, Findings, Applications. DPA Helsinki Investigations III edited by ERKKI KOMULAINEN and PERTTI KANSANEN. May 1981. 52 pp.

No. 57 Psychometric Test Theory and Cognitive Processes: A Theoretical Scrutiny and Empirical Research by JARKKO LEINO. October 1981, 74 pp.

x) Out of print.